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Spatial Models of Political Competition Under Plurality Rule: A Survey of Some Explanations of the Number of Candidates and the Positions They Take

by

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ABSTRACT This paper surveys work that uses spatial models of political competition to explain the number of candidates and the positions that they take in plurality rule elections.

1 Introduction

Our economic lives are heavily influenced by the actions of individuals and bodies whom we elect. The policies of national governments have major effects on much economic activity; school boards influence our decisions about how to spend a significant fraction of our incomes; and the chairs and committees of academic departments may decide the criteria on which to base the year-to-year changes in our salaries. What explains the relation between the policies pursued by an elected body and the voters' preferences? The models that I survey are designed to address this question for the electoral system of plurality rule.

Much of the work in the field aims to explain a small number of features of electoral outcomes; it is around these features that I organize the discussion. I

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take the position that the purpose of formal models is to check the coherence and consistency of ideas that enhance our understanding of the phenomena that we observe, and not, for example, to construct models that necessarily have "realistic" assumptions. I further claim that results are not interesting unless they can be given clear informal interpretations—that is, unless they can be confirmed by intuition. Consequently, for each of the main models that I discuss I try to express in simple terms the main idea that it captures.

The work that I discuss is designed to explain various aspects of the stylized fact that in plurality rule electoral systems there are usually two major parties with similar, but not identical, positions. (I do not discuss the many other phenomena that have been studied using the spatial model.) For each aspect of this observation that I discuss, I isolate the main ideas that have been proposed as explanations, describe how these ideas have been formalized, and consider how robust the formal models are to perturbations of the assumptions. I include statements of results and proofs whenever they can be given in a reasonable amount of space, since an understanding of a formal proof is often necessary in order to appreciate the limitations of a result.

In Section 2 I describe the basic spatial model, show how it provides an explanation of the observation that participants in two-candidate elections often choose similar positions, and discuss the robustness of this explanation. In Section 3 I describe explanations of the fact that participants in two-candidate contests generally do not adopt the *same* position. Finally, in Section 4 I discuss formalizations of ideas for explaining why plurality rule appears to lead to two-party competition. A more detailed outline of the paper is given in Figure 1.

I do not intend my method of organization to imply that every model should originate as an attempt to illuminate a specific phenomenon; to ask the question "what happens in a model in which the voters are imperfectly informed about the candidates' positions?" may be just as useful a starting point as the question "how can we explain the fact that candidates in two-candidate competitions adopt similar positions?". The former question sug-

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Figure 1: An outline of the paper.

gests that an alternative way of organizing the material is to group models according to their characteristics: for example, perfect versus imperfect information; fixed number of candidates versus endogenous determination of set of candidates. Several other surveys are organized along these lines (for example Calvert (1986), Coughlin (1990b), and Shepsle (1991)).

2 Why do the participants in two-candidate elections choose similar positions?

2.1 The Main Idea: Hotelling's Model

Most of the models that I discuss are set in the following framework. There is a set X of political *positions* or *policies*, a set I of *citizens*, and a set N of *candidates* or *parties*. Each candidate chooses a position (i.e. member of X), and then each citizen chooses which, if any, of the candidates for whom to vote. Given the votes that are cast, an electoral mechanism selects the candidate who is the winner.

This framework was suggested by Hotelling (1929). His specific model, which is the basis of most of the work that I discuss, captures the following idea.

> In a two-candidate competition each candidate can obtain more votes by moving closer to the other candidate, so that a situation is stable only if the candidates' positions are the same.

The model is the following. The set X of positions is one-dimensional, identified with the real line $\mathbb{R}$, the set I of citizens is a continuum, and the set N of candidates is finite, say $\{1, \ldots, n\}$. The (complete, transitive, reflexive) preference relation $\succsim_i$ over X of each citizen i is continuous and has the property that there is a position $\hat{x}_i \in X$—the *ideal position* of citizen i—such that

$$x \succsim_i y \text{ if and only if } |x - \hat{x}_i| \leq |y - \hat{x}_i|. \tag{1}$$

Under this assumption the preference relation $\succsim_i$ can be represented by a utility function that is increasing on $(-\infty, \hat{x}_i)$, decreasing on $(\hat{x}_i, \infty)$, and symmetric about $\hat{x}_i$. For this reason a preference relation that satisfies the condition is referred to as *single-peaked* and *symmetric*.

The assumption of single-peakedness is central to the model—it is what distinguishes the spatial formulation; it means that the citizens are in basic agreement about the meaning of points in the set X. They disagree about which is the most desirable point but concur that the real line orders policies on each side of their ideal points in the same way. Though often taken for granted, the assumption imposes considerable structure on preferences. If there are three candidates, for example, then for *no* single-peaked preferences is the position of the middle candidate the worst. (By contrast, the assumption of symmetry is made only for convenience; it is not essential (see Section 2.4).)

The assumption of single-peaked preferences implies that if there are two candidates, one at x and one at $y > x$, then *all* citizens with ideal points less than x prefer the candidate at x and all those with ideal points greater than y prefer the candidate at y. The assumption of symmetry further implies that the citizens who prefer the candidate at x are precisely those with ideal points less than $(x + y)/2$.

A second central assumption of Hotelling's model is that every citizen votes, endorsing the candidate whom she likes best. That is, voting is *sincere*; the citizens are not players in the game. Given the symmetry of the preferences, this assumption implies that each citizen votes for the candidate whose position is closest to her ideal point, so that the fraction of the votes received by each candidate can conveniently be represented in a diagram like Figure 2.

A third basic assumption of the model is that each candidate cares only about the outcome of the election—the profile of votes received by the candidates—and, unlike the citizens, not about the position of the winning candidate. A number of specific objectives for candidates have been used in the literature; to establish the basic result in Hotelling's model that I now present, only a very weak assumption on preferences is required, namely that each candidate prefers to win than to tie for first place, and prefers to tie than to lose. (Note that an objective often used in the literature—vote maximization—is,

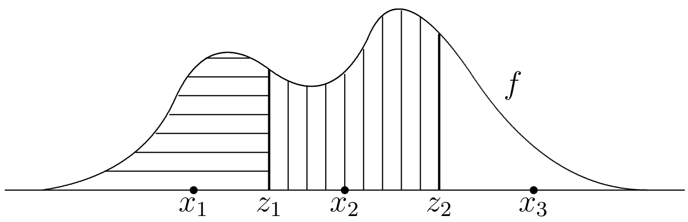

Figure 2: Dividing up the votes in Hotelling's model. The horizontal axis is the policy space X and the function f is the density of ideal points. There are three candidates, with positions x_1, x_2, and x_3; z_1 is the midpoint of $[x_1, x_2]$ and z_2 is the midpoint of $[x_2, x_3]$. The fraction of the votes received by candidate 1 is equal to the area shaded by horizontal lines, the fraction received by candidate 2 is equal to the area shaded by vertical lines, and the fraction received by candidate 3 is equal to the remaining area.

in the presence of more than two candidates, inconsistent with this natural condition (see the discussion in Section 2.9.1).)

To state this restriction precisely I need the following definitions. Given a profile $x \in X^n$ of positions for the candidates let $v_j(x)$ be the fraction of citizens who vote for candidate j, under the assumption that if there are many candidates at the same point then they share the votes for that point equally, and let $M_j(x)$ be the plurality of candidate j:

$$M_j(x) = v_j(x) - \max_{k \neq j} v_k(x).$$

The assumption on the preference relation $\succsim_j$ of each candidate j is then the following:

$$x \succ_j y \succ_j z \text{ whenever } M_j(x) > 0,\ M_j(y) = 0, \text{ and } M_j(z) < 0. \tag{2}$$

(Note that I have not explicitly specified an "outcome" of the election. It is enough for my current purposes to assume that each candidate's preferences satisfy (2); the outcome that lies behind this might be that the candidate who

- Policy space X is one-dimensional.
- Fixed finite set of candidates.
- Each candidate cares only about winning; she prefers to win than to tie for first place, and to tie than to lose.
- Continuum of citizens, each of whom has symmetric single-peaked preferences over X.
- Candidates simultaneously choose positions in X.
- Knowing the candidates' positions, every citizen votes, and does so sincerely.
- Each candidate is perfectly informed about the citizens' preferences.

Figure 3: The main assumptions of Hotelling's model.

receives the most votes becomes a dictator, or that she merely becomes the most powerful member of a legislature that may include other candidates.)

A final basic assumption is that each candidate is perfectly informed about the citizens' preferences.

In summary, **Hotelling's model** is the strategic game[3] $\langle N, (A_j), (\succsim_j)\rangle$ in which $A_j = X$ for each $j \in N$, the preference relation $\succsim_j$ of each candidate $j \in N$ satisfies (2), and the preference relation $\succsim_i$ of each citizen i that lies behind the M_j's is single-peaked and symmetric (i.e. satisfies (1)). The main assumptions of the model are summarized in Figure 3. I refer to the variant of this model in which each candidate may choose not to enter the competition—that is, the action set of each player is $X \cup \{Out\}$ rather than X—as **Hotelling's model with exit**.

When there are two candidates Hotelling's model yields the following result, a formal expression of the idea given at the beginning of the section. Let F be

[3]Throughout I use the terminology and notation of Osborne and Rubinstein (1994).

the distribution function of ideal points (so that $F(x)$ is the fraction of citizens whose ideal point is at most x). Assume that the support of F is an interval, so that there is a unique position $m \in X$ such that $F(m) = \frac{1}{2}$: the *median* of F. The result shows that we can deduce not only that the equilibrium positions of the candidates are the same but also that they coincide with the median of F.

Proposition 1. *If there are two candidates ($n = 2$) then Hotelling's model $\langle N, (X), (\succsim_j)\rangle$ (in which the candidates' preferences satisfy (2) and the citizens' preferences are single-peaked and symmetric (i.e. satisfy (1))) has a unique Nash equilibrium, in which the position chosen by each candidate is the median of the distribution of the citizens' ideal points.*

Proof. First note that a candidate can ensure that she ties for first place by choosing the same position as the other candidate. By (2) a candidate prefers to tie for first place than to lose, so there is no equilibrium in which a candidate loses. It follows that in any equilibrium the candidates tie for first place.

Let the position chosen by candidate j be x_j for $j = 1, 2$. If either $x_1 \neq x_2$ (in which case neither x_1 nor x_2 is the median of F, else the candidates do not tie) or $x_1 = x_2 \neq m$ then either of the candidates can win outright by moving to the median. By (2) she prefers to win outright than to tie for first place, so that we have $x_1 = x_2 = m$ in any equilibrium.

Finally, it is immediate that $(x_1, x_2) = (m, m)$ is an equilibrium. □

The same result clearly survives in Hotelling's model with exit so long as each candidate prefers to tie for first place with one other candidate than to choose *Out*.

Hotelling was primarily interested in a more complex economic model in which the players choose prices for their products, as well as locations, though he recognized that the model in which prices are absent gives us insights into the nature of the positions chosen by politicians (1929, pp. 54–55). Downs (1957, especially Ch. 8) used Hotelling's model to study political equilibrium extensively; for this reason the model is sometimes referred to as the "Hotelling–Downs model". (Black (1958), reporting on work done in the 1940's, also studied the model, but was concerned mainly with issues that I do not treat here.)

How robust is this result? I first consider the assumptions in Figure 3 one by one in the case that there are just two candidates; I start with the assumptions that are not crucial, then turn to those that are. Subsequently I consider the effect of allowing for more than two candidates.

2.2 Multidimensional Policy Space

Hotelling (1929, pp. 55–56) recognized that his main idea—the fact that in a two-candidate competition each candidate has an incentive to move towards the other—applies to the case in which the policy space X is multi-dimensional. Suppose, for example, that all the citizens' preferences are symmetric, $X \subset \mathbb{R}^2$, and the candidates' positions are $x_1 \in X$ and $x_2 \in X$. Let L be the line through the midpoint of the line segment $[x_1, x_2]$ that is perpendicular to this line segment. Then the citizens who vote for candidate 1 are those whose ideal points lie on the same side of L as x_1. If candidate 1 moves closer to candidate 2 then the line moves closer to y and candidate 1 attracts more votes. (Davis and Hinich (1966, 1967, 1968) were the first to explore this case formally.)

However, in this case there is not in general a position that is an analog of the median in the one-dimensional case: only exceptionally is there a position with the property that *every* hyperplane through it divides the distribution of ideal points into two equal parts. If such a position exists then there is an equilibrium in which both candidates choose that position. (The significance of such a position was first noted by Davis, DeGroot, and Hinich (1972).) If not then there is no equilibrium (for any point there is always another point that attracts more than half of the votes). (There is a Nash equilibrium in which both candidates choose the same position x if and only if x is a "Condorcet equilibrium", a notion from the theory of voting. An early paper that examines the conditions under which such an equilibrium exists is Plott (1967); for a survey of subsequent results see Austen-Smith (1983, Section 2).)

So far I have restricted the candidates to use pure strategies. Kramer (1978) shows that there is an equilibrium in mixed strategies if each citizen's preference relation is represented by a strictly quasi-concave function, the distribution of citizens' preferences is continuous, and each candidate maximizes the same continuous function of her plurality. McKelvey and Ordeshook (1976) examine the size of the supports of any equilibrium mixed strategies; when $X \subset \mathbb{R}^k$ they show that there is an equilibrium in which the support of each candidate's mixed strategy is a subset of the convex hull of the set of points each of which is the intersection of k median hyperplanes (though there may exist equilibria in which the strategies have supports outside this set).

If the assumption of sincere voting is replaced by an assumption that allows citizens to abstain and relates their behavior probabilistically to the payoffs they obtain from the candidates' positions, according to an exogenously specified function, then under some conditions a pure strategy equilibrium exists in the multidimensional model (see Hinich, Ledyard, and Ordeshook (1972, 1973) and Slutsky (1975)). In some cases it may be possible to interpret the probabilities that are assumed in this approach to be the candidates' beliefs about the rational behavior of citizens whose characteristics the candidates do not know; see Section 2.8.

In summary, Hotelling's basic idea survives when the space of policies is multidimensional, though in this case a pure strategy equilibrium may not exist.

2.3 Citizens' Preferences That Are Not Single-Peaked

As remarked above, the assumption that the citizens' preferences are single-peaked is an essential part of the spatial formulation. If arbitrary preferences are allowed then the spatial structure is irrelevant, but there may be classes of preferences that have enough structure to yield non-trivial results. It seems, for example, that small deviations from single-peakedness may allow equilibria to exist in which the candidates adopt significantly different positions. I know of no systematic analysis of this case.

2.4 Citizens' Preferences That Are Not Symmetric

The assumption that the citizens' preference relations are symmetric in Proposition 1 can be replaced by an assumption that the degree of asymmetry is bounded across the citizens (that is, there is no sequence of citizens with preferences whose degree of asymmetry is increasing without bound). (In the absence of the assumption it could be that *all* the citizens with ideal points in (x, y) vote for, say, the candidate at x, so that there is an equilibrium in which one candidate's position is the median and the other candidate's position is some other point.)

2.5 Variations in Timing

Suppose that, rather than choosing their positions simultaneously, the candidates either move in a fixed order or may choose positions whenever they wish. Each of these games has a unique equilibrium that coincides with that of the simultaneous-move game: both candidates choose the median ideal point.

2.6 Electing a Legislature

In Hotelling's model each candidate is concerned only about winning the single election in which she is involved. In many cases, however, a single election is only part of a collection of elections that determines the composition of a legislature. Consider the case of a legislature that contains an odd number of members, each of whom is elected in a separate district; suppose that the distribution of the citizens' ideal points is not the same in all districts.

Assume first that there are two parties, each of which fields a candidate in each district. Further assume that all the candidates of each party must adopt the same position and each candidate prefers an outcome in which her party has a majority in the legislature with probability one half and she wins her seat with probability one half to the outcome in which her party definitely has a minority in the legislature but she definitely wins her seat. Then all the candidates of each party agree on the position that their party should adopt; the only equilibrium is that in which both parties adopt the same position, equal to the median of the collection of medians of the distributions of ideal points in the districts. (This observation is due to Hinich and Ordeshook (1974).) The argument is the following: if one of the parties adopts a slightly different position then it wins a minority ($(k-1)/2$ out of k) of the seats in the legislature; if both parties choose the same position then each wins a majority with probability $\frac{1}{2}$.) That is, the basic insight of Hotelling's model survives: the pressure on parties to win leads the parties to adopt the same positions.

Now assume that each candidate is free to choose any position she wishes, the policy that is carried out by the party in the event that it wins a majority of seats being some aggregate of the candidates' positions that is sensitive to all the candidates' positions and is known to the citizens. Then under the assumptions above about the candidates' preferences the outcome in which each candidate adopts the median ideal position in the median district is again the unique equilibrium. By the same token, if some of the candidates' preferences are reversed then it is no longer an equilibrium for all candidates to adopt the median of the median positions.

Another case, examined by Austen-Smith (1984), is that in which each candidate cares exclusively about her own fortunes. If in this case each candidate is free to choose any position and the party position is some aggregate of the candidates' positions that is sensitive to all the candidates' positions, then it is not an equilibrium for all candidates to adopt the median of the median ideal positions, since by moving a little to the left, and hence moving her party position a little to the left, a candidate in a left-of-center district can ensure that she wins outright, rather than tying with the candidate of the other party. In fact, if in this case a candidate can *always* move her party's position some minimal amount in one direction by moving her own position one unit in that direction then there is no equilibrium at all. To see this, suppose that the position of party A in some district j is different from the median position m_j in that district. Then, exactly as in the proof of Proposition 1, the candidate of party A in this district can move enough that her party's position becomes m_j and hence she wins outright. We conclude that the party's positions must coincide with the median position in every district, which is not possible since these medians differ.

Austen-Smith generates an equilibrium in this case by adopting the not unreasonable assumption that the extent to which a candidate can affect the position of her party is limited—beyond some point further moves to extreme positions are discounted and may cause the party position to move in the *opposite* direction. He shows that if each candidate cares about the number of votes that she receives then in any equilibrium the positions of the *parties* coincide. The argument is again that of the proof of Proposition 1: by changing her position a candidate can move the position of her party closer to that of the other party and hence increase the number of votes that she receives. The positions of the candidates in any given district may not coincide, though this appears to be of limited significance since these positions are only instruments used by the candidates to affect party policy. (Nevertheless, the model does offer an explanation for difference in candidates' positions at the district level.)

Rather than assuming that each of the candidates is *a priori* affiliated with a party, we can assume that the candidates choose positions, then form parties of the basis of the similarity of these positions. Austen-Smith (1986) takes this approach. The problem of coalition-formation is difficult; game theory currently offers no clear solution. Austen-Smith assumes that citizens have beliefs about the probability of each possible coalition forming, depending on the positions adopted and the size of the coalition (that is, coalition-formation in his model is a black box). The issues that arise are complex, with the consequence that beyond proving existence of an equilibrium the analytical results are limited. The models of Austen-Smith and Banks (1988) and Baron (1993) are related, though their focus is different. In these models three parties contest an election in which the outcome is determined by proportional representation (putting the models beyond the scope of this survey); the process of coalition-formation is specified explicitly. A number of features of the equilibria in the models are of interest, though it is not clear to what extent they depend on the details of the formulations.

As this brief discussion illustrates there are many possibilities for modeling systems of elections that select a legislature. (Austen-Smith (1989) is a survey of work in the area.) Compared to the significance of the topic and range of questions that remains to be answered the amount of work that has been done so far is small. Some examples have been studied, but no general results have so far emerged. The forces that lie behind Proposition 1 play a role in the models, though clearly there are also other principles at work.

2.7 Candidates Who Care About the Policy Enacted

In Hotelling's model each candidate cares only about winning the election. Consider now the consequence of assuming, to the contrary, that each candidate j has a fixed "ideological stance" (ideal position) x_j^* and cares *only* about how close the policy of the winner of the election is to this position. For simplicity assume that these preference relations, like those of the citizens, are symmetric. That is, for each candidate j the preference relation $\succsim_j$ over profiles x of positions for which there is a unique winner $w(x)$ satisfies

$$x \succsim_j y \text{ if and only if } |x_{w(x)} - x_j^*| \leq |y_{w(y)} - x_j^*|. \tag{3}$$

If there is more than one candidate tied for first place in the profile x then each candidate evaluates the induced lottery over winning positions according to the expected value of some (not necessarily quasi-concave) function that represents her preferences over profiles with a unique winner.

Each candidate may choose any policy she wishes, as before; having taken a position a candidate is committed to implement it if elected. The following result shows that if the candidates' stances are on opposite sides of the median ideal position then, despite their ideological attachments, the candidates have an incentive to satisfy the whims of the median voter: the basic idea behind Proposition 1 holds, as the following result shows (see Wittman (1977, Proposition 5; 1990, Section 7), Calvert (1985, p. 75), and Roemer (1991, Theorem 2.1)).

Proposition 2. *Consider the variant of Hotelling's model in which each candidate's preference relation satisfies (3) (instead of (2)) (and each citizen's preference relation is single-peaked and symmetric (see (1)), as in Hotelling's model). If there are two candidates ($n = 2$) and $x_1^* \leq x_2^*$ then the Nash equilibria of this model are as follows, where m is the median of the distribution of the citizens' ideal points.*

- *if $x_1^* \leq m \leq x_2^*$ then $(x_1, x_2) = (m, m)$ is the unique Nash equilibrium*
- *if $x_1^* \leq x_2^* < m$ then (x_1, x_2) is a Nash equilibrium if and only if either $x_2^* \leq x_1 = x_2 \leq m$, or $x_1 \leq x_2 = x_2^*$.*

Proof. First suppose that $x_1^* \leq m \leq x_2^*$. If both x_1 and x_2 are on the same side of m and $x_1 \neq x_2$ then the winner can move slightly closer to her favorite position and still win; if $x_1 = x_2 < m$ then candidate 2 can move slightly closer to her favorite position and win outright and if $x_1 = x_2 > m$ then candidate 1 has a similar profitable deviation. If the candidates are on opposite sides of m and one wins outright then by moving to the median the loser can win outright; she prefers the median policy to that of the other candidate. Finally, if the candidates are on opposite sides of m and tie for first place then by moving any small amount $\epsilon > 0$ closer to m either candidate can win; for ϵ small enough she prefers obtaining her new position with certainty than obtaining her old position with probability $\frac{1}{2}$ and that of her opponent with probability $\frac{1}{2}$. The only remaining possibility is that $(x_1, x_2) = (m, m)$, which is indeed an equilibrium.

Now suppose that $x_1^* \leq x_2^* < m$. I first show that in any equilibrium we have $x_2 \in [x_2^*, m]$. Suppose that $x_2 < x_2^*$. If either $x_1 \leq x_2^*$, or $x_1 > x_2^*$ and candidate 1 loses or ties for first place, then candidate 2 can increase her payoff by moving to x_2^*. If $x_1 > x_2^*$ and candidate 1 wins then candidate 1 can increase her payoff by moving to x_1^*. Now suppose that $x_2 > m$. If $x_1 < x_2^*$ then candidate 2 can increase her payoff by moving to x_2^*; if $x_1 \geq x_2^*$ and candidate 1 wins then candidate 1 can increase her payoff by moving slightly to the left; and if $x_1 \geq x_2^*$ and candidate 2 wins or ties for first place then candidate 2 can increase her payoff by moving slightly to the left. Now, if $x_2 \in (x_2^*, m]$ then we must have $x_1 = x_2$ (otherwise the winning candidate, if one candidate wins outright, or else the rightmost candidate who ties for first place, can increase her payoff by moving slightly to the left); if $x_2 = x_2^*$ then we need $x_1 \leq x_2$, otherwise candidate 1 can increase her payoff by moving to x_2^*. Finally, any pair (x_1, x_2) in which either $x_2^* \leq x_1 = x_2 \leq m$ or $x_1 \leq x_2 = x_2^*$ is clearly an equilibrium. $\square$

This result shows that the basic idea behind Hotelling's model holds even if each candidate, like each citizen, cares about the policy enacted, rather than about whether or not she wins. However, if we modify also the informational assumption of Hotelling's model by assuming that the candidates are uncertain of the distribution of the citizens' ideal points then we find that equilibria in which the candidates take different positions are possible; this case is discussed in Section 3. Further, if the parties cannot perfectly commit to carry out the policies they announce then also the logic of the result is disturbed; this case is considered in Section 3.1.2.

The candidates' preferences in the model in this section are specified exogenously; they are unrelated to the preferences of the citizens who may support them. New issues arise if the candidates are drawn from the set of citizens; I discuss models in which this is so in Sections 3.1.3 and 4.3.

2.8 Strategic Voting

Each citizen in Hotelling's model is not a rational actor but merely an automaton who votes for her favorite candidate. How robust are the conclusions of Hotelling's model to this assumption?

The natural extension of Hotelling's model is the extensive game in which first the candidates simultaneously choose positions (as in Hotelling's game), then, knowing these positions, every citizen chooses whether to vote, and, if so, for which candidate. In this case it is convenient to assume that there is a finite number of citizens, rather than a continuum as in Hotelling's model; throughout I assume that $I = \{1, \ldots, \ell\}$. The most interesting results arise when it is costly for each citizen to vote, and all the players (the citizens and the candidates) are uncertain about the citizens' characteristics (their voting costs and ideal points). This model was first studied by Ledyard (1981, 1984); I refer to it as the **Hotelling–Ledyard model**. Precisely, the model is a Bayesian extensive game with observable actions $\langle \Gamma, (\Theta_i), (p_i), (U_i) \rangle$ (see Osborne and Rubinstein (1994, Section 12.3)) in which

- the set of players in Γ is $I \cup N$ (where $I = \{1, \ldots, \ell\}$ and $N = \{1, \ldots, n\}$) and Γ is the extensive game form in which first the candidates (members of N) simultaneously choose positions (points in X), then, informed of these positions, the citizens (members of I) simultaneously choose whether to vote and, if so, for whom.

- $\Theta_i = X \times C$ for some $C \subseteq \mathbb{R}$ for each $i \in I$, and Θ_i is a singleton for each $i \in N$.

- for each $i \in I$ we have $U_i(\theta, z) = u_{x_i}(x^*(z)) - c_i$ if i votes, and $U_i(\theta, z) = u_{x_i}(x^*(z))$ if she does not, where x_i is i's ideal point, c_i is her voting cost, and $x^*(z)$ is the position of the winner of the election when the terminal history of Γ is z. For each $i \in N$ the function U_i is generated by preferences that satisfy (2).

- Policy space X is one-dimensional.
- Fixed finite set of candidates.
- Each candidate cares only about winning; she prefers to win than to tie for first place, and to tie than to lose.
- Finite number of citizens, each of whom has symmetric single-peaked preferences over X.
- Candidates simultaneously choose positions in X.
- After observing the candidates' positions, every citizen chooses whether or not to vote, and, if so, for which candidate. If citizen i votes then she incurs the cost c_i.
- The voting cost and ideal position of each citizen may or may not be private information.

Figure 4: The main assumptions of the Hotelling–Ledyard model.

Note that the form of each p_i, the probability measure on Θ_i that characterizes the uncertainty about citizen i's characteristics, is not specified. The main assumptions of this model are given in Figure 4 (cf. Figure 3). (Ledyard (1981, 1984) makes specific assumptions about each p_i; I use the term "Hotelling–Ledyard model" to refer to games in which these specific assumptions are not necessarily satisfied.) As in the case of the Hotelling model, I sometimes consider a variant that I refer to as the **Hotelling–Ledyard model with exit**, in which each candidate has the option of not running in the election (that is, the action set of each candidate is $X \cup \{Out\}$ rather than X).

In the remainder of this section (2.8) I restrict attention to the case in which there are two candidates ($n = 2$).

To analyze the model it is convenient to begin by considering the Bayesian games in which the citizens are involved once the candidates choose their positions. I refer to these games as voting subgames (though they are not subgames of the extensive game associated with the Bayesian extensive game with observable actions unless there is perfect information). Formally, a *voting subgame* is a Bayesian game $\langle I, \Omega, (A_i), (T_i), (\tau_i), (\hat{p}_i), (\succsim_i)\rangle$ in which

- the set of states of nature is $\Omega = (X \times C)^\ell$ (the set of all profiles $\{(x_i, c_i)\}_{i\in I}$, where x_i is the ideal position of citizen i and c_i is her voting cost)
- the set A_i of actions of citizen i is $\{1, 2\} \cup \{0\}$ (where the action $j \in \{1, 2\}$ means "vote for j" and the action 0 means "do not vote")
- the set T_i of possible types of citizen i is $X \times C$.
- the signal function τ_i of citizen i is defined by $\tau_i((x_1, c_1), \ldots, (x_\ell, c_\ell)) = (x_i, c_i)$ (i.e. citizen i is informed only of her own characteristics)
- the belief $\hat{p}_i$ of citizen i is obtained from the probability measures (p_j) of the Bayesian extensive game
- the preference relation $\succsim_i$ of citizen i over lotteries over $(\times_{i\in I}A_i) \times \Omega$ is defined by the expected value of her payoff function in the Bayesian extensive game.

2.8.1 Costless Voting under Perfect Information

A simple case to begin with is that in which voting is costless for all citizens ($c_i = 0$ for all $i \in I$) and the citizens' ideal points are known. In this case every voting subgame has many Nash equilibria. In particular, *any* citizen behavior in which the numbers of votes received by the candidates differ by at least two is a Nash equilibrium, since in such a case an individual who changes her behavior has no effect on the outcome. An implication is that the full two-stage game has many subgame perfect equilibria, including ones in which the candidates adopt different positions.

However, at least in the case in which there are just two candidates, the size of the set of equilibrium outcomes is dramatically reduced if we require that each citizen use a *(weakly) undominated* strategy, since it is a (weakly) dominant strategy for any citizen to vote for her favorite candidate. (If the candidates' positions are different and the number of votes received by a citizen's favorite candidate is either equal to or one less than the number of votes received by the other candidate then voting for her favorite candidate leads to an outcome that the citizen prefers to that which results when she either abstains or votes for the other candidate; otherwise the citizen's action has no effect on the outcome.) Thus if there are two candidates then in any Nash equilibrium of a voting subgame in which every citizen's strategy is undominated, voting is sincere. (This result depends on the fact that there are just two candidates; see Section 2.9 below.)

It follows that if a subgame perfect equilibrium in which the citizens are restricted to use undominated strategies exists in this version of the Hotelling–Ledyard model then the position of each candidate is the median ideal point. If the candidates' preferences satisfy (2) then there is an equilibrium of this sort (although at such an equilibrium one of the candidates may lose, since the citizens are indifferent between the two candidates and hence may split their votes arbitrarily between them). In the Hotelling–Ledyard model with exit under the same assumptions there are equilibria in which either one or both of the candidates enter at the median. That is, removing the restriction that citizens vote sincerely in Hotelling's model, while retaining the assumptions of perfect information and costless voting, has little effect on the set of equilibria of the game.

2.8.2 Costly Voting under Imperfect Information

If voting is costly then an entirely different picture emerges. In this case a citizen votes (for her preferred candidate) only if the expected benefit from doing so exceeds her cost; the expected benefit depends on the probability that the citizen's vote affects the outcome and on the citizen's utility difference between the candidates' positions. (I continue to assume that there are just two candidates.) The fact that citizens may abstain modifies the incentive for a candidate to move closer to her rival that is at the heart of the Hotelling model: a candidate who does so may *lose* the votes of some citizens who no longer find the difference between the candidates large enough to make it worthwhile to vote. What are the implications for the equilibria of the game?

A case that is convenient to work with is that in which the citizens' voting costs may differ and are drawn independently from the same continuously differentiable distribution H, each citizen knowing her own voting cost but not that of any other citizen. Under this assumption each citizen is *a priori* identical as far as her voting cost is concerned. It simplifies the analysis to assume also that each citizen is *a priori* identical as far as her ideal position is concerned. That is, rather than assuming that the distribution of ideal points is known, assume (following Ledyard (1981, 1984)) that each citizen's ideal position is drawn independently from the same continuously differentiable distribution G (independent of H), and each citizen knows her own ideal position but not that of any other citizen. Under these assumptions each citizen is *a priori* identical in every respect, and the knowledge of her own characteristics (ideal point and voting cost) conveys no information about the other citizens' characteristics.

I begin by considering the equilibria of the voting subgames. Restrict attention to *symmetric* Nash equilibria of these games, in which two citizens with the same characteristics take the same action. Such an equilibrium is given by a function $\alpha\colon X \times C \to \{1,2\} \cup \{0\}$ that associates an action with each each pair consisting of an ideal point and a voting cost, with the property that for each $(x,c) \in X \times C$ the action $\alpha(x,c)$ is optimal for a citizen with characteristic (x,c) given that the other citizens' behavior is determined by α and the citizen's belief about the distribution of characteristics.

When is it optimal for a citizen with characteristic (x,c) to vote for candidate j? Suppose that $u_x(x_1) > u_x(x_2)$ (she prefers the position of candidate 1). Then her optimal action is either to vote for candidate 1 or to abstain. Her vote makes a difference to the outcome only if the other citizens either cast the same number of votes for each candidate (in which case her vote makes candidate 1 win outright rather than tie for first place) or cast one less vote for candidate 1 than for candidate 2 (in which case her vote makes candidate 1 tie for first place rather than lose). In both cases the increase in her payoff that the more desirable outcome yields is the same (equal to $\frac{1}{2}[u_x(x_1) - u_x(x_2)]$), so we need to find only the probability of either of the events occurring. To do so, let $q_j(\alpha)$ be the probability, as determined by α, that a random citizen votes for candidate j: that is, $q_j(\alpha)$ is the probability that (x, c) takes a value for which $\alpha(x, c) = j$. Then the probability of either of the two events in which the vote of a citizen who prefers candidate 1 to candidate 2 is decisive is

$$\begin{aligned} p_1(\alpha) &= \sum_{k=0}^{[n/2]} \frac{n!}{k!k!(n-2k)!} q_1^k q_2^k (1 - q_1 - q_2)^{n-2k} + \\ & \sum_{k=0}^{[(n-1)/2]} \frac{n!}{(k+1)!k!(n-2k-1)!} q_1^k q_2^{k+1} (1 - q_1 - q_2)^{n-2k-1}, \end{aligned} \tag{4}$$

where $[x]$ denotes the largest integer less than or equal to x and for clarity I have written q_i rather than $q_i(\alpha)$. (The candidates tie when k citizens vote for each of them, for any possible value of k.) Thus, given α, the expected gain in payoff from voting for candidate 1 rather than abstaining for a citizen who prefers candidate 1 to candidate 2 is $\frac{1}{2}p_1(\alpha)[u_x(x_1) - u_x(x_2)]$; it is hence optimal for such a citizen with voting cost c to vote for candidate 1 if

$$c \leq \tfrac{1}{2}p_1(\alpha)[u_x(x_1) - u_x(x_2)]. \tag{5}$$

In summary, α is an equilibrium if for each pair (x, c) we have $\alpha(x, c) = 1$ whenever (5) is satisfied with strict inequality and only when it is satisfied with weak inequality, $\alpha(x, c) = 2$ under a symmetric condition, and otherwise $\alpha(x, c) = 0$.

Now suppose (again following Ledyard (1981, 1984)) that all possible voting costs are nonnegative ($C \subseteq \mathbb{R}_+$) and that the distribution H is continuous and has support $[0, \overline{c}]$ for some $\overline{c}$. Further assume that each payoff function u_x is symmetric about x. Then we have $u_x(x_1) = u_x(x_2)$ for a citizen with ideal point $x = (x_1 + x_2)/2$, so that the fraction of such citizens who vote is zero (since this is the fraction with voting cost 0). Some of the citizens with other ideal points may vote, depending on the nature of their payoff functions. Two cases to which I refer later are the following.

Concave payoff functions If u_x is concave then the difference between $u_x(x_1)$ and $u_x(x_2)$ *increases* the further the citizen's ideal point x is from the midpoint of x_1 and x_2. That is, extremists care intensely about the differences between moderate candidates.

Convex payoff functions If u_x is convex on each side of x then the difference between $u_x(x_1)$ and $u_x(x_2)$ is largest when x is close to x_1 or x_2; extremists care little about the differences between moderate candidates.

The assumption of concavity is often adopted, first because it is associated with "risk aversion" and second because it makes it easier to show that an equilibrium exists. However, I am uncomfortable with the implication of concavity that extremists are highly sensitive to differences between moderate candidates (a view that seems to be shared by Downs (1957, pp. 119–120)). Perhaps the Republican and Democratic parties in the US are run by people whose opinions are extreme relative to those of the average voter for these parties (Tim Feddersen has made this point to me), but does Tony Benn really perceive a huge difference between Margaret Thatcher and Enoch Powell?[4] Further, it is not clear that evidence that people are risk averse in economic decision-making has any relevance here. I conclude that in the absence of any convincing empirical evidence it is not clear which of the assumptions is more appropriate.

For each of these assumptions, possible forms of the optimal behavior of a citizen as a function of her characteristic (x, c) are shown in Figure 5, taking

[4] David Laidler suggested this specific example.

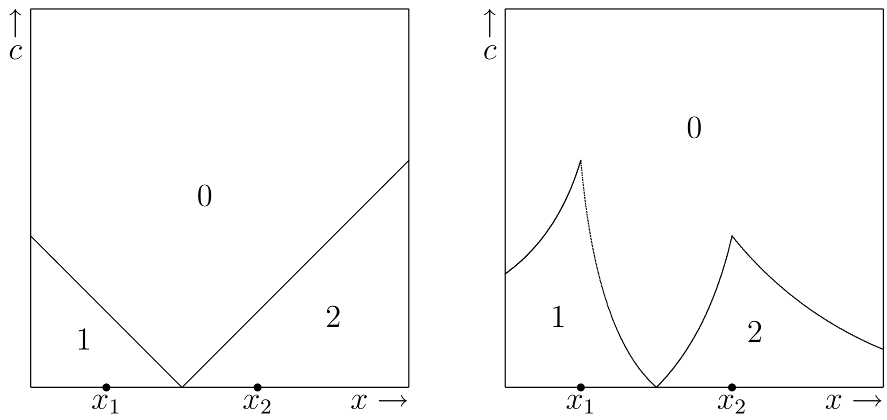

Figure 5: The optimal voting decision of a citizen as a function of her characteristic (x, c), given the probabilities p_1 and p_2 of a vote being pivotal. The case in the left-hand panel could arise if the citizen's payoff function is concave; that in the right-hand panel could arise if her payoff function is convex on each side of her ideal point.

as given the probabilities p_1 and p_2 that a vote is pivotal in favor of either of the candidates.

To consider the extent to which the basic idea captured by Hotelling's model survives, suppose that the candidates' positions x_1 and x_2 are different and the voting subgame has an equilibrium in which some citizens vote. Consider the effect of candidate 1 moving her position a little closer to x_2. By doing so she reduces the amount that any citizen gains by voting for her in the event the citizen's vote is decisive. On this account she *diminishes* her support: given the fractions of citizens who vote for each candidate (and hence the probability of a citizen's vote being decisive) some of the citizens who previously voted for her will now find it not worthwhile to do so. However, this effect is mitigated by two factors:

- In reducing the difference between her platform and that of the other candidate she also reduces the incentive for citizens to vote for her rival, and hence reduces the support for the rival too.

- Even if a small move towards her rival reduces her support relative to that of the rival, a large move, to exactly the same position as the rival, leads to an equilibrium of the voting subgame in which she ties for first place (no one votes, since the positions are the same).

Further, the incentive in the Hotelling model does not lose its force completely: by moving closer to x_2 candidate 1 gains the votes of some of those citizens who previously voted for candidate 2. Nevertheless, for some distributions H and G there may be an equilibrium in which the candidates choose different positions (suppose that G is symmetric and bimodal, and suppose that x_1 and x_2 are at the modes), though no example exists in the literature and it is not clear that there is one that is robust.

However, even when the incentive for the candidates to converge is still dominant, the common position that the candidates choose in equilibrium no longer bears any necessary relation to the median of the distribution of ideal points. To see this, suppose that every function u_x is concave, the number of citizens is large, and there is an equilibrium in which both candidates adopt the position x^*. In this equilibrium no citizen votes and the candidates tie for first place. Since the number of citizens is large a candidate maximizes her probability of winning by maximizing her expected plurality; for equilibrium we require that a candidate who differentiates herself from her rival does not increase this expected plurality.

Suppose that candidate 1 moves her position x_1 slightly to the left of x^*. Then some citizens find it worthwhile to vote, as in the left-hand panel of Figure 5; all these citizens' voting costs are small (given that x_1 is close to x^*). If there are citizens with arbitrarily small voting costs (i.e. if $H'(0) > 0$) then for a small change in x_1 the fraction of those with ideal position $x < x^*$ who vote for candidate 1 is proportional to $-u'_x(x^*)$ (given that each citizen's payoff is linear in the voting cost). Similarly the fraction of those with ideal position $x > x^*$ who vote for candidate 2 is proportional to $u'_x(x^*)$. Thus the change in candidate 1's expected plurality is proportional to

$$\int_X u'_x(x^*)g(x)dx,$$

where g is the density of G. In an equilibrium this must be zero, so that the common position x^* of the candidates maximizes $\int_X u_x(x^*)g(x)dx$ (which is concave). This informal argument suggests the following result, due to Ledyard (1984, Theorem 1).

Proposition 3. *Consider the Hotelling–Ledyard model in which there are two candidates ($n = 2$), each citizen's voting cost is drawn independently from the distribution H, and each citizen's ideal position is drawn independently from the distribution G (independently of H). Suppose that H is continuously differentiable with support $[0, \overline{c}]$ for some $\overline{c} > 0$, and the density of voting costs is positive at zero ($H'(0) > 0$). Suppose also that G is continuously differentiable and that u_x is continuously differentiable and strictly concave for all $x \in X$. Then in all perfect Bayesian equilibria of the game in which the equilibrium in each voting subgame is symmetric, both candidates choose the position x^* that maximizes $\int_X u_x(x^*)g(x)dx$ (and no citizen votes).*

If for all $x \in X$ we have $u_x(y) = -|y - x|$ for all y then the maximizer of $\int_X u_x(y)g(x)dx$ is the median of G, but for other utility functions it generally differs from the median. If, for example, $u_x(y) = -(y - x)^2$ for all x and y then the maximizer is the *mean* of G.

Thus even in cases in which costly strategic voting under imperfect information leads to an equilibrium in which (as in Hotelling's model) the candidates' positions are the same, this common position in general differs from the median. Note that since a small move by a candidate away from the common equilibrium position attracts citizens with very small voting costs, the characteristics of these citizens are crucial in determining the nature of the equilibrium. If voting cost is correlated with ideal position, for example, the characterization of the equilibrium is different from that given in Proposition 3.

The fact that the median loses significance in situations in which it is not the case that every citizen certainly votes for her favorite candidate was first recognized by Hinich (1977, 1978). He imposes no rationality on the citizens' choices, however. Rather he takes the function that gives the probability that any citizen votes for a particular candidate as the primitive, following Hinich, Ledyard, and Ordeshook (1972). (Work in the "probabilistic voting" framework is surveyed in Coughlin (1990a, 1992).) As Slutsky (1975) argues, the fact that rationality is not imposed on the citizens' behavior is problematic; it is hard to know if the forms of the probability functions assumed can be rationalized as the outcome of reasoned choices by the citizens unless one builds a model like Ledyard's in which voting behavior is included explicitly. (Since the price (in terms of analytical complexity) of building rational voting behavior into the model is high, this argument does not imply that work in the probabilistic voting framework is not useful.)

In proposing a model in which voting behavior is rational Slutsky (1975) rejects the idea upon which the Hotelling–Ledyard model is based—namely that a citizen is motivated to vote because there is a positive probability that she thereby affects the outcome. The basis of his argument is that this probability is negligible. Certainly in the *equilibrium* of Proposition 3 this is not so: the candidates' positions are the same, so no one votes and, by voting, any citizen can, with certainty, affect the candidate who is elected. Even in voting subgames in which the candidates' positions are different and the number of citizens is large, there are circumstances in which the probability p_i that a single vote affects the outcome in favor of candidate i (either by creating or breaking a tie) is not negligible. If, for example, there are 10,000 citizens and each votes for each candidate with probability 0.2 (i.e. $n = 10,000$ and $q_1 = q_2 = 0.2$ in (5)) then this probability exceeds 0.01. However, if $q_1 \neq q_2$—if the candidates are not tied in the polls—then the probability of a single voter being pivotal is smaller, and decreases rapidly as q_1 and q_2 diverge (see Chamberlain and Rothschild (1981)). Nevertheless, whether or not the probability of a single voter being pivotal is high enough to justify voting behavior remains unclear. First, the assumption that all citizens have the same probability of voting for any given candidate may be a poor model of voter perceptions that underestimates the probability of a vote being pivotal.[5] Second, even a probability of 0.001 could be significant given a relatively small cost of voting and a perceived large potential gain (how much more will candidate 1 raise your taxes than candidate 2?). As the number of voters increases the probability goes to zero (as shown under slightly different assumptions by Palfrey and Rosenthal (1985)); however for relevant values of the parameters the probability may still be large enough to play an important role.[6]

The feature of the equilibrium in Proposition 3 that is unattractive is not that the probability of a citizen's vote influencing the outcome is too small, but that it is too large: it is one, since no one votes. In order to obtain an equilibrium in which some citizens vote an incentive must be introduced for the candidates to adopt different positions. Models that contain such incentives are considered in Section 3.

2.9 More Than Two Candidates

If there are more than two candidates then the players' incentives in the Hotelling and Hotelling–Ledyard models change in the following two significant ways.

- A candidate may no longer increase the number of votes that she receives by moving her position closer to that of some other candidate. If, for example, the distribution of ideal points has a concave density and candidate 1's position x_1 is to the left of the mode while candidate 2's

[5] Perhaps a better model is one in which each candidate has a pool of supporters that is roughly the same size and is relatively committed to the candidate and there is a pool of undecided voters. I am grateful to Jeffrey S. Rosenthal for helpful discussions on this point.

[6] The statement of Palfrey and Rosenthal (1985, p. 72) that "in very large electorates ... both criticial cost levels are approximately zero", while literally correct, may be misleading; their conclusion "In other words, voters will not vote in large elections if the net cost is positive" seems to be unwarranted.

position x_2 is to the right then the best position in $[x_1, x_2]$ for candidate 3 is in the interior of this interval.

- It is no longer a dominant strategy for a citizen to vote for her favorite candidate. If the winner of the election is very likely to be either candidate 2 or candidate 3 then a citizen who prefers the position of candidate 1 to the other two positions is better off voting for either candidate 2 or candidate 3 (whichever she prefers).

2.9.1 Sincere Voting

To see the extent to which Hotelling's basic insight survives despite these differences in incentives, first consider what happens if we assume that there are $n \geq 3$ candidates rather than two in the basic Hotelling model.

For the case in which there are two candidates I imposed only the weak restriction (2) on the candidates' preferences. A standard approach in the literature is to assume that each candidate's preference relation is represented by a specific function. A function that is sometimes used is borrowed from the economic version of Hotelling's model: each candidate wishes to maximize the number of votes that she receives. This is consistent with the very natural restriction (2) on preferences when there are two candidates, but is inconsistent with this restriction when there are more candidates: a candidate who wins outright may, if she moves her position closer to that of a neighbor, increase the number of votes that she receives but at the same time increase the number of votes received by her other neighbor enough that she is no longer the outright winner. In many elections losers are not powerless, but I take it to be a basic feature of political competition that candidates prefer to win than to lose, and thus argue that the criterion of vote maximization is not sensible.

One objective that seems appropriate for some political competitions is plurality maximization: each candidate prefers an outcome in which her margin of winning is greater (or her margin of loss is less). Preferences that are represented by this objective are continuous: winning by a small margin is preferred only a little to losing by a small margin. At the opposite end of the spectrum are preferences in which a candidate prefers to stay out of the competition than to lose, whatever the margin of loss. In both cases the Hotelling model has no (pure) Nash equilibrium under a wide range of circumstances (Osborne (1993)). Consider, for example, the second case. Assume that the preference relation $\succsim_j$ of each candidate j over profiles of positions satisfies

$$x \succ_j y \text{ whenever } x_j = \textit{Out} \text{ and } M_j(y) < 0 \tag{6}$$

and

$$x \succ_j y \text{ whenever } M_j(x) = 0,\ w(x) = 2, \text{ and } y_j = \textit{Out}, \tag{7}$$

where $w(x)$ is the number of candidates with a nonnegative plurality in x. The following result is taken from Osborne (1993).

Proposition 4. *If there are at least three candidates, whose preferences satisfy (6) and (7) in addition to (2), then for almost any distribution of ideal points Hotelling's model with exit (in which each citizen's preference relation is single-peaked and symmetric (see (1))) has no Nash equilibrium in pure strategies.*

Outline of proof. Suppose that there is an equilibrium in which k candidates enter; let y_1 be the leftmost occupied position. The argument proceeds in steps.

Step 1. Every candidate who enters receives the fraction $1/k$ of the votes. (If any candidate receives less then she loses and prefers to stay out.)

Step 2. There are at least two candidates at y_1. (A lone candidate could move closer to her neighbor and (using Step 1) win outright.)

Step 3. There are exactly two candidates at y_1. (If there were three or more then any one could move slightly to one side, obtain more than $1/k$ of the votes, and (using Step 1) win outright.)

Step 4. We have $y_1 = F^{-1}(1/k)$. (If y_1 were larger then either of the candidates at y_1 could move slightly to the left and win outright; if it were smaller then one of the candidates could move slightly to the right and win outright.)

Step 5. The second occupied position from the left is given by $y_2 = F^{-1}(1/k) + 2(F^{-1}(2/k) - F^{-1}(1/k))$. (The midpoint of $[y_1, y_2]$ is $F^{-1}(2/k)$ by Steps 1, 3, and 4.)

Step 6. For almost any distribution F of ideal points there can be only one candidate at y_2. (By the argument in Step 3 there are at most two candidates at y_2; by the argument in Step 4 there can be two only if $y_2 = F^{-1}(3/k)$, which is not so for almost any F.)

Step 7. By a similar argument there can be only one candidate at every occupied position to the right of y_2.

Step 8. There are two candidates at the rightmost occupied position (by the argument in Step 3).

Since Steps 7 and 8 are contradictory, the result follows. $\square$

For special distributions of ideal points a Nash equilibrium does exist. For example, Cox (1987b) finds equilibria in the case that this distribution is uniform and every candidate is a plurality maximizer.

Presumably the game has mixed strategy equilibria under the assumptions of Proposition 4. However, it is unlikely that these equilibria are tractable. I conclude that a straightforward extension of Hotelling's model to the case of more than two candidates gives us little insight into the outcome of multi-candidate competition.

2.9.2 Strategic Voting

If voting is strategic then the situation is quite different, as Feddersen *et al.* (1990) demonstrate. They study the Hotelling–Ledyard model with exit when voting is costless, there is perfect information, and the citizens' payoff functions are concave; the solution they use is a variant of subgame perfect equilibrium in which the action of each voter after any history is undominated. The main idea that drives their result is the following. Suppose that there is an equilibrium in which three or more candidates enter and choose different positions. Since a candidate enters only if she has a positive probability of winning all those who enter must receive the same number of votes. This has two significant consequences: (a) the outcome of the election is the lottery in which the probability of each candidate winning is the same, and (b) every voter is pivotal. But now by the concavity of the citizens' payoff functions and (a), any citizen who votes for one of the extreme candidates prefers the positions of at least one of the *other* candidates to the (probabilistic) outcome of the election;[7] by (b) any citizen who deviates by voting for one of those preferred candidates induces the outcome in which that candidate is the certain winner of the election, an outcome that the citizen prefers to the lottery over all entrants. We conclude that there is no equilibrium in which three or more positions are occupied. The same argument rules out any equilibrium in which three or more candidates enter, so long as they do not all choose the same position. If two candidates enter then all citizens vote sincerely in equilibrium, and the incentive for convergence in the Hotelling model with two candidates takes over. Thus there is no equilibrium in which two candidates enter at distinct positions.

We conclude that the only configuration that may be induced by an equilibrium is that in which all candidates who enter choose the same position. Such a configuration is indeed an equilibrium if the common position is the median ideal point. The equilibrium is supported by the following equilibria in the voting subgames: if a candidate deviates from the median then all the citizens who prefer the median to the deviant's position vote for *one* of the candidates at the median. That is, in the event of a deviation the voting behavior of the citizens is coordinated. (Feddersen *et al.* (1990, p. 1014) remark that the need for such coordination is a "disturbing feature" of the equilibria.)

To make these arguments precise, suppose that the preferences of candi-

[7] As Feddersen et al. point out this argument does not rest heavily on concavity. If the citizens' ideal points are distributed densely enough over X then there is a citizen who prefers the positions of at least two candidates to the (probabilistic) outcome of the election even if no citizen's payoff function is concave.

date i are represented by the payoff function

$$\pi_i(x) = \begin{cases} 0 & \text{if } x_i = Out \\ -c & \text{if } x_i \in X \text{ and } i \notin W(x) \\ b/\omega - c & \text{if } x_i \in X,\ i \in W(x), \text{ and } |W(x)| = \omega, \end{cases}$$

where c is the cost of entry, b is the benefit of winning outright, and $W(x)$ is the set of candidates with a nonnegative plurality (the set of winners of the election) when the profile of positions is x. The result[8] of Feddersen *et al.* (1990) is the following.

Proposition 5. *Suppose that the number ℓ of citizens is odd, the preference relation over X of each citizen is represented by a concave payoff function, the policy space X is $[0, 1]$, voting is costless, and there is perfect information in the Hotelling–Ledyard model with exit. A profile of actions for the candidates is induced by a subgame perfect equilibrium in which each citizen's action in every voting subgame is undominated if and only if ω candidates enter for some $\omega \in [1, b/c]$, ℓ/ω is an integer, and the position of every entrant is the median of the distribution of the citizens' ideal points.*

Outline of proof. The argument consists of the following steps.

Step 1. All candidates who enter receive the same number of votes (otherwise one loses, and would be better off not entering), so that ℓ/ω is an integer and every voter is pivotal.

Step 2. Either all candidates choose the same position or just two candidates enter and choose different positions (by the argument in the text preceding the statement of the result).

Step 3. In any voting subgame in which just two positions are occupied every citizen votes for one of the candidates at the position that she prefers (since this is the only undominated action).

[8] Feddersen et al. adopt the additional assumption that citizens do not have the option of abstaining. Feddersen has pointed out to me that this assumption is unnecessary.

Step 4. If two candidates enter and do so at different positions then either of them can deviate by moving closer to the median and win outright (using Step 3).

Step 5. If all the candidates who enter choose the same position then this position is the median ideal point (since if not then by Step 3 a candidate can deviate to the median and win outright).

Step 6. No more than b/c candidates enter (since if more enter then the expected payoff to each of them is negative).

Step 7. For any $\omega \in [1, b/c]$ for which ℓ/ω is an integer there is an equilibrium in which ω candidates enter at the median ideal point. (In the voting subgame that follows a deviation by a candidate all citizens who prefer the median to the position of the deviant vote for one of the candidates (say the one with the lowest index) who remains at the median.) $\square$

This result demonstrates that modeling citizens as rational actors has a significant effect on the equilibria of the model. However, it appears to depend crucially on the assumption of perfect information, which makes every voter pivotal in any equilibrium. If the citizens and candidates are uncertain of the median of the distribution of ideal points then the equilibrium appears to collapse; the form of any equilibrium in this case is is not clear. The informational structure of Ledyard (1984), discussed in Section 2.8.2, is natural, but it is not clear how tractable Ledyard's model is when there are more than two candidates. Myerson and Weber (1993) suggest a much simpler model, in which the citizens' uncertainties about each other are not modeled explicitly, but rather the existence of such uncertainty is used to motivate the (exogenous) probabilities $\{p_{kj}\}$ that each citizen assigns to her vote causing the winner of the election to be some candidate j rather than another candidate k. These pivot probabilities are not allowed to be arbitrary: they are shared by all citizens, and if candidate j receives fewer votes than candidate k then the probability p_{hj} of a single vote changing the winner of the election from some candidate h to j is of a smaller order of magnitude than the probability p_{hk} of a single voter changing the winner from h to k. Given the pivot probabilities, citizens are assumed to vote rationally, as in the Hotelling–Ledyard model. Myerson and Weber give an example of the implications of such a model of voting for a candidate positioning game in which there are three candidates. However, in the example the candidates do not have the option of staying out of the competition, so that the equilibrium is not directly comparable to that of Proposition 5; the robustness of this result to the presence of a little imperfect information remains unclear.

2.10 Summary

The basic insight afforded by Hotelling's model—that there is an incentive for candidates in two-candidate competitions to adopt similar positions—is rather robust. However, when citizens are treated as rational decision-makers for whom voting is costly the common position chosen by the two candidates is no longer necessarily the median of the distribution of ideal points, as it is in the standard Hotelling model.

When there are more than two (potential) candidates then the basic incentive inherent in the Hotelling model is significantly diluted. If information is perfect and voting is costless then Proposition 5 shows that enough of the incentive survives to lead all candidates who enter the competition to choose the same position, but whether or not this result survives in the presence of imperfect information is unclear.

3 But candidates don't choose the *SAME* positions

Even if candidates for office frequently adopt similar positions, they rarely adopt exactly the *same* position. As we have seen, however, in Hotelling's model with two candidates the incentive for the candidates to converge overwhelms any reason they might have to differentiate themselves. What could lead them to do so?

3.1 Policy-Motivated Candidates

3.1.1 The Basic Model

One idea is that candidates care about the policy that is enacted, not just about winning *per se*.

Candidates who care about the policy that is enacted and disagree about the most desirable policy have an incentive to offer different policies.

To consider how this idea might be formalized, suppose that two candidates have ideological positions on opposite sides of the median ideal point and that they are currently offering different policies, both closer to the median ideal point than their favorite positions. As one candidate moves her policy closer to that of the other candidate she becomes worse off in the event that she wins (since her policy is further from her favorite) but at the same time may increase her probability of winning. Thus potentially she faces a tradeoff, the existence of which could result in an equilibrium in which the candidates' positions are different. However, as we saw above (Section 2.7) there is *no* such equilibrium if there is perfect information. The reason is that the change in the probability of winning as one candidate moves her position closer to that of the other candidate is very abrupt. In particular, if the position of one candidate is different from the median then there is always a position for the other candidate that wins with probability one. As we saw (Proposition 2), this leads to a unique equilibrium, in which both candidates choose the median policy.

One way to modify the model in order to capture the idea is to add some uncertainty.[9] Assume that there are two candidates, who are uncertain about the distribution of the citizens' ideal points. Specifically, assume that there is a family $\{F_\gamma\}$ of distribution functions of ideal points, indexed by the parame-

[9] After completing this paper I became aware of Lindbeck and Weibull (1993), which takes a different route: it adds to the basic model the assumption that each voter has a "party identity", which biases her vote in favor of one of the candidates.

ter γ, with γ equal to the median of F_γ. Suppose that both candidates believe that the distribution function of γ is the nonatomic distribution K. If the pair of positions chosen by the candidates is $x = (x_1, x_2)$ with $x_1 < x_2$ then the probability that candidate 1 wins is $\pi_1(x) = K((x_1 + x_2)/2)$ (the probability that γ is less than $(x_1 + x_2)/2)$ and the probability that candidate 2 wins is $\pi_2(x) = 1 - \pi_1(x)$. Hence each candidate now faces a smooth tradeoff as she moves her position closer to that of her rival. Suppose that candidate j's preferences over pairs x of positions are represented by the function

$$\sum_{k=1}^{2} p_k(x) U_j(x_k), \tag{8}$$

where U_j is a real-valued single-peaked function on X for which the maximizer is x_j^*. Then we have the following result.

Proposition 6. *Consider the variant of Hotelling's model in which there are two candidates, each of whom*

- *cares about the policy enacted, her payoff function being given by (8), with* $x_1^* \neq x_2^*$
- *is uncertain about the distribution of the citizens' ideal points, believing the distribution of the median ideal point to be nonatomic.*

In any Nash equilibrium of this model the policies proposed by the candidates are different.

Proof. If $x_1 = x_2 = x^*$ then the policy x^* is enacted with probability one and at least one candidate's favorite position—say candidate 1's favorite position—differs from x^*. Then if candidate 1 moves to a position between x^* and x_1^* at which her probability of winning is positive, she is better off, since she obtains the more desirable position with positive probability. And there is such a position, since the distribution of the median ideal point is nonatomic. □

(Wittman (1983, Proposition 2) and Hansson and Stuart (1984, Theorem 1) establish results like this, but take the functions $p_j(x)$ as primitives, rather than deriving them from the candidates' uncertainty about the median ideal point. The approach here is due to Roemer (1991), who also (Theorem 4.1) gives conditions under which an equilibrium exists; see also Roemer (1993).)

3.1.2 Variation: Commitment and Information

If candidates care about the policy that is enacted and there is a temporal gulf between the election and the enactment of the policy the question arises to what extent a candidate is committed to a policy that she proposes. Suppose that we model the situation as a three-stage game: first the candidates propose policies; then the citizens vote; finally the winning candidate enacts a policy. If neither party can commit to a policy and the cost of an announcement is independent of its relation to a candidate's favorite policy then announcements contain no information; in any subgame perfect equilibrium the citizens ignore the announcements and the winning party enacts its favorite policy in the last stage of the game, regardless of the policy it proposed. Thus the outcome is that the candidate whose policy is favored by the most citizens wins and enacts that policy: there is no convergence at all. (This observation is due to Alesina (1988).)

If the election is one of a sequence in which the politicians are engaged then an elected candidate may have an incentive not simply to carry out her favorite policy, even if she can make no formal commitment, since current actions have implications for the future behavior of other players. Alesina (1988) uses the "folk theorem" from the theory of repeated games (see, for example, Osborne and Rubinstein (1994, Chapter 8)) to make this point in a model in which the politicians are infinitely-lived: outcomes that are not Nash equilibria of the one-shot game can be supported in an infinitely repeated game if the players use strategies that "punish" each other for deviations.

Alesina and Spear (1988) and Harrington (1992c) further pursue this point in a model in which there is an infinite sequence of *finitely*-lived politicians. Alesina and Spear argue that there are mechanisms that transfer payoff between present and future incumbents of the same party; possible future incumbents, who care about their chances of winning, have an incentive to reward current incumbents for catering to the whims of the electorate rather than to their own impulses.

Harrington models a different idea: if a politician cares not only about the policy that is enacted while she is in office but also about the policy that is carried out after she leaves office then she has an incentive to take actions that enhance the chances that she will be succeeded by a member of her own party. If voters believe that a party's past behavior is indicative of its future behavior then it can be optimal for a current incumbent to moderate her policies away from her own favorite policy in order to increase the chance that her successor will be a member of her own party (and thus carry out a policy more to her own liking than the policy that would be carried out by a member of the rival party). Harrington's model has the following interesting consequence. Suppose that a politician's preferences put some weight on winning the election and some weight k on the policy pursued by the winner. Since a retired politician has no possibility of winning she cares *only* about the policy pursued by the winner, however small k is. Thus even a politician who is almost entirely office-motivated may be induced while in office to carry out a policy that diverges from that of her rival, even though a politician who is *completely* office-motivated wants to offer the same policy as her rival. This result contrasts with a result of Calvert (1985) that in the static model considered in Proposition 6 the outcome is continuous in the degree of office-motivation of the candidates.

In the models considered so far the policy enacted by one politician has no direct connection with the policies that a subsequent office-holder can enact. Phelan (1991) formalizes the idea that if policies can be changed only slowly—if there is some inertia in the system—then a candidate who cares about the policy enacted and has a positive probability of not being reelected may have an incentive to enact a policy that is more *extreme* than her favorite policy. If there is either no inertia or complete inertia then the incentive is absent, but between these two extremes it can be advantageous to adopt extreme policies that can be only partially dismantled by subsequent incumbents.

Rosenthal (1982) studies a different dynamic model, in which candidates inherit positions that they have limited powers to change. Uncertain of whom she will face as rivals in the future, each candidate chooses a position bearing in mind not only its desirability as a competitor for the position of her next rival, but also the flexibility it gives her in the more distant future. Equilibria may have a number of interesting features. One example is that the candidates' equilibrium positions may not be dominant if it is sufficiently disadvantageous to move one's position a little, even though a substantial move, which must be accomplished in several small steps, is desirable (see Rosenthal's Example 3). Another example is that poor positions may be adopted because they provide particularly good opportunities for future movement (Rosenthal's Example 4).

Banks (1990) and Harrington (1992b) (see also Harrington (1992a)) study variants of the three-stage announcement–voting–enactment game in which the citizens do not know the true positions of the candidates. The issue is the extent to which the candidates' announcements are informative of their true positions.

Banks assumes that it is costly for candidates to implement policies that differ from those they announce. He finds that if the cost increases fast enough with the extent of divergence between the announced and implemented policies then in an equilibrium the announcements of candidates with extreme positions identify the candidates' positions, while those of moderates do not. One consequence is that if both candidates are moderates then each is elected half of the time, which means that half of the time a candidate favored by a minority of the voters is elected.

Harrington's idea is different. He assumes that the candidates' announcements have no direct implications for their subsequent payoffs. Rather, his key assumption is that the policy that an office-holder can carry out depends on the preferences of the citizens: she cannot simply carry out her favorite policy regardless of the citizens' preferences. A consequence is that a candidate has some incentive to announce her true policy preference, since if she does so then whenever she wins, her true position is supported by a majority of voters, so that she is more likely to be able to implement that position. The result is that in some cases there is an equilibrium in which the candidates make truthful announcements.

Although not concerned directly with the convergence of candidates' positions, some of the issues that arise in Austen-Smith and Banks (1989) are similar. They study the extent to which future elections act as a discipline device on current candidates, an issue first raised by Barro (1973) and subsequently studied by Ferejohn (1986), among others. (The main issue that this literature addresses is beyond the scope of this survey.) The candidates in their model incur costs not because they are induced to carry out policies different from their favorites (as in the models above), but because they are induced to expend effort to affect the legislative outcome. There is a single voter, who observes only a stochastic function of this effort. In a two-period model both candidates exert no effort at all in the second period, and so the citizen is indifferent between them; the vote cast in the second period can thus depend on the behavior of the candidates in the first period. Austen-Smith and Banks study the consequence of the citizen using one of a family of specific second-period voting strategies that reward first period incumbents who carry out policies close to those that they announce. They find that there is an equilibrium in which both candidates are induced to exert the best effort level for the voter. That is, despite the imperfect information, there is complete convergence in the candidates' first period actions.

3.1.3 Variation: Endogenous Parties

In the model of Proposition 6 the parties' preferences are given exogenously; they are not derived from the preferences of the parties' supporters. What can we conclude from a model in which parties are composed of the citizens who support them? If the citizens' payoff functions are concave then those with extreme ideal points are more sensitive to differences between candidates' positions. Thus if running a party is costly it will be carried out by extremists. This leads to the following idea.

> If citizens' payoff functions are concave then each party will tend to be run by extremists, who have an interest in making the party position extreme.

This idea is formalized by Feddersen (1992, 1993), who removes the strategic parties from the Hotelling–Ledyard model, leaving only the citizens as players. Each citizen may be inactive or vote, at a cost, for any position; the position that receives the most votes wins. That is, the strategic game is that in which the set of players is the set I of citizens and the action set of each player is $X \cup \{Abstain\}$, where the action $x \in X$ of citizen i is that of voting for position x. The preferences of each player i are given as follows. For any profile x of actions let $W(x)$ be the set of winning positions (i.e. $W(x)$ is the set of positions $y \in X$ for which $|\{i \in I\colon x_i = y\}|$ is maximal). Then player i's payoff to the profile x is

$$\begin{cases} \sum_{y \in W(x)} u_i(y)/|W(x)| - c & \text{if } x_i \in X \\ \sum_{y \in W(x)} u_i(y)/|W(x)| & \text{if } x_i = Abstain, \end{cases} \tag{9}$$

where $u_i\colon X \to \mathbb{R}$ and $c > 0$.

By Proposition 7 below, at most two positions receive votes in any Nash equilibrium of this game. These positions cannot be very close since then no citizen finds it worthwhile to vote. However, if the cost of voting is not large then the minimal separation of the winning positions in a Nash equilibrium is relatively small (Feddersen (1992)).

The notion of Nash equilibrium in this model allows parties to form—that is, in an equilibrium sets of citizens all vote for the same position—but it does not allow them to change their positions as they do in the Hotelling model, since only deviations by *single* citizens are considered. To put back actions by parties into the model Feddersen (1993) studies the implications of the notion of coalition-proof Nash equilibrium (CPNE), which requires that the outcome be robust to deviations by *sets* of citizens that are themselves immune to further deviations. The set of CPNE outcomes is much smaller than the set of Nash equilibrium outcomes, and in every such outcome there is some separation between the parties. Above we saw that, in the presence of perfect information, no separation is predicted by the variant of the Hotelling model in which candidates care about the policy enacted. So what accounts for the separation in Feddersen's model? The point is that a situation is an equilibrium if no group of citizens can deviate and vote for some other position, *given* the voting behavior of all the other citizens. In contrast, in the Hotelling model a party finds it profitable to deviate if it increases its payoff by so doing, assuming that the citizens *react* to the deviation. Thus in Feddersen's simultaneous-move model a group of citizens supporting some position that is closer to the median than the ideal position of any of its members finds it advantageous to move that position *further* from the position of its rival since such a move *does not affect* the number of votes that the party receives and results in a more desirable outcome for all the party members in the event that the position wins.

In summary, Feddersen's model captures the idea above, but does so at the price of assuming that the strategic reasoning of the players in the game is short-sighted: the players who support a party do not anticipate that a change in their party position will affect the voting behavior of the other citizens. Perhaps a multistage model, in which party formation and voting behavior are divorced, would better capture parties' strategic calculations. At the same time, Feddersen's model improves upon the model of Proposition 6 in that it endogenizes the motivations of the parties.

3.2 Uncertainty by Voters About the Candidates' Positions

Suppose that it is not possible for a candidate to convey precisely to the citizens the position that she takes: each citizen perceives each candidate's position to be a probability distribution over X. If the candidates' positions are perceived to differ in their riskiness then, since a risk averse citizen prefers the less risky platform when choosing between two candidates with the same expected policy, it seems that there may be a reason for candidates to separate their positions.

A candidate may lose votes as she moves her position too close to that of her rival if citizens are less certain about her position than that of her rival.

To see how this idea can be formalized consider the following example. Candidate i's position is μ_i but is perceived by the citizens to be a random variable $\mathbf{x}_i$ with mean μ_i and variance σ_i^2; assume that $\mu_1 < \mu_2$ and $\sigma_1^2 < \sigma_2^2$. All citizens' payoff functions are quadratic: the preferences of a citizen with ideal point $\hat{x}$ are represented by the function $-(\hat{x} - x)^2$. Then since $\mathrm{E}[-(\hat{x} - \mathbf{x})^2] = -(\hat{x} - \mathrm{E}(\mathbf{x}))^2 - \mathrm{V}(\mathbf{x})$ for any random variable $\mathbf{x}$ (where $\mathrm{V}(\mathbf{x})$ is the variance of $\mathbf{x}$) a citizen with ideal point $\hat{x}$ votes for candidate 1 if

$$-(\hat{x} - \mu_2)^2 - \sigma_2^2 < -(\hat{x} - \mu_1)^2 - \sigma_1^2,$$

or

$$\hat{x} < \frac{\sigma_2^2 - \sigma_1^2}{2(\mu_2 - \mu_1)} + \frac{\mu_1 + \mu_2}{2}.$$

(In particular, the ideal point of the citizen who is indifferent between the two candidates exceeds the mean of the candidates' positions.) It follows that candidate 1 increases the number of votes she receives as she moves closer to candidate 2, but the same is *not* true of candidate 2: as she moves her position closer to that of candidate 1 the number of votes that she receives first increases but then decreases, as the fact that her position is more uncertain than that of candidate 1 starts to outweigh in the minds of the risk-averse voters the fact that her *mean* position is preferable. (The value of μ_2 that maximizes the number of votes received by candidate 2 is $\mu_1 + \sqrt{\sigma_2^2 - \sigma_1^2}$.)

A consequence of this analysis is that if the variances of the candidates' positions are different then in this example there is no (pure) Nash equilibrium (though presumably there is a mixed strategy equilibrium); the example suggests that nonexistence is a general phenomenon, though I know of no results to that effect. Hug (1992) shows that if there are three parties then for some values of the variances there are pure strategy equilibria in which the candidates adopt different positions.

Although the two-candidate model lacks an equilibrium if the candidates' choices are made simultaneously, there is a subgame perfect equilibrium in the sequential game in which the candidate whose position has lower variance moves first. In this equilibrium the low-variance candidate chooses the median ideal point and the high-variance candidate chooses a best response, which differs from the median; the low variance candidate is the outright winner. Bernhardt and Ingberman (1985) exploit the fact that the sequential game has a subgame perfect equilibrium in a model in which the difference between the variances of the candidates' positions is based on their statuses as incumbent and challenger. The incumbent's position is not very risky if it is similar to her past position, but increases in riskiness as it diverges from this past position. The challenger's position has a fixed degree of riskiness, which is less than that of an incumbent who adopts the same position as previously. In an equilibrium the candidates adopt different positions.

In summary, although a pure strategy equilibrium fails to exist in a simultaneous move model the idea that candidates whose positions are imperfectly perceived have an incentive to adopt different positions can be captured in a model of sequential choice. The drawbacks of such a model are twofold: the sequential structure is left unexplained and the fact that the candidate with the higher variance always loses raises the question of why she wishes to participate in the competition. To address these issues it seems that a richer model is needed.

So far I have assumed that the relation between the policy chosen by a candidate and the random variable that the citizens perceive is exogenous. There is some work that examines the case in which the candidates can *choose* to make their positions unclear. The starting point is a result of Shepsle (1972) that if the citizens are risk-averse in Hotelling's model with two candidates then there is no equilibrium in which the candidates choose to be ambiguous: there is no mixed strategy equilibrium. If the policy space is multi-dimensional then there *is* a mixed strategy equilibrium (see Section 2.2); the same is presumably true if there are more than two candidates (though I know of no analysis of this case). Thus Shepsle's result appears not to be robust. Other work that offers explanations of ambiguity includes Alesina and Cukierman (1990) and Glazer (1990); see also Harrington (1992b, Section 6).

3.3 Separation to Mitigate the Effect of Entry

The equilibrium in Hotelling's model with two candidates, in which both candidates choose the median ideal position, is highly vulnerable to entry: a third candidate can enter at any point close enough to the median and win outright. This suggests that the presence of a potential entrant may induce the candidates to adopt different positions:

> Candidates have an incentive to separate their positions in order to minimize the effect of the entry of further candidates.

This idea is formalized by Palfrey (1984) in a model in which two established parties first choose their positions simultaneously, then a third party chooses its position. (Brams and Straffin (1982) earlier studied the optimal positions of entrants in response to the given positions of two existing candidates.) Voting is sincere and each candidate's payoff is the expected number of votes that she receives. Palfrey shows that in a subgame perfect equilibrium the two established parties choose distinct positions; the third party chooses a position between the two established parties. If, for example, the distribution of the citizens' ideal points is uniform on $[0, 1]$ then there is a subgame perfect equilibrium in which the two established parties choose the positions $\frac{1}{4}$ and $\frac{3}{4}$ and the third party enters at $\frac{1}{2}$; if either of the established parties deviates and adopts a position closer to the median $(\frac{1}{2})$ the entrant maximizes the number of votes she receives by locating at a point a little more extreme than the position of the deviant, causing the latter to obtain fewer votes than previously.

Palfrey's model clearly captures the idea described above. How sensitive is the equilibrium to his assumptions? First suppose that we modify the timing of the candidates' choices. Weber (1992b) shows that if the two established parties choose their positions sequentially rather than simultaneously then the equilibria of the game remain essentially the same. If we relax the restrictions on timing completely, allowing all three candidates to choose their positions whenever they wish then also it seems that Palfrey's equilibrium survives. Thus the asymmetric structure of Palfrey's model does not appear to play a major role in his result.

Now consider the effect of modifying the nature of the candidates' preferences. I argued in Section 2.9 that the assumption that a candidate aims to maximize the number of votes she receives is unattractive, especially when there are more than two candidates. More reasonable objectives for a participant in a plurality-rule election are the maximization of her probability of winning or the maximization of her plurality. If each candidate maximizes her plurality, while the remaining structure of Palfrey's model is retained, then it seems that his result does not qualitatively change. If the distribution of ideal points is uniform, for example, there is a subgame perfect equilibrium in which the first two candidates choose the positions $\frac{3}{10}$ and $\frac{7}{10}$ and the third candidate chooses $\frac{1}{2}$. (The plurality of the third candidate in this situation is $-\frac{2}{10}$; if she locates instead just to the left of $\frac{3}{10}$ then her plurality is less than $-\frac{2}{10}$.)

If, however, each candidate is concerned only about winning the election, has the option of staying out of the competition, and prefers to do so than to enter and lose, then the nature of the equilibrium completely changes. In this case Palfrey's game has a subgame perfect equilibrium in which one of the first two candidates enters at the median ideal position, the other stays out of the competition, and the third candidate also enters at the median. The same pattern of choices is a subgame perfect equilibrium outcome of the game in which all three candidates move sequentially. Further, it seems that in neither of these cases is there an equilibrium in which two candidates choose different positions. If all three candidates are treated symmetrically and may act whenever they wish then in every equilibrium exactly *one* candidate enters (Osborne (1993)). In this case a second candidate refrains from entering because if she does so then a third candidate can enter and win outright, causing the second entrant to lose. (Note that in an equilibrium of Palfrey's model the third party always loses.)

Finally, consider the effect of increasing the number of potential candidates. The only case that has been studied, to my knowledge, assumes that each candidate cares only about winning the election and has the option of staying out of the competition; the potential candidates may act whenever they wish. Then there is an equilibrium with exactly $n - 2$ entrants (at the median) if there are $n = 4$ or $n = 5$ potential entrants (Osborne (1993)).

In conclusion, the separation that Palfrey finds in his model appears to be robust to changes in the sequential structure of the game though not to modifications of the candidates' objectives: his equilibrium does not survive when the candidates care only about winning the election and have the option of not entering, in which case there are equilibria in which a single candidate enters. However, it is possible that the equilibrium in this latter case is sensitive to the assumption of perfect information: there may be equilibria in which there is separation if the candidates are uncertain about the characteristics of the voters.

4 Why are there two parties?

Almost all of the discussion above relates to models in which there are just two candidates. One of the most widely cited stylized facts is that under systems of plurality rule there are indeed two main parties. (This observation was first made by Duverger (1954, p. 206–280), and is referred to in the literature as "Duverger's Law". Riker (1982) and Cox (1991) survey research on the topic and Wright and Riker (1989) contains some systematic evidence on it.) What can explain this stylized fact? I discuss three ideas.

4.1 Votes for Minor Parties Are Wasted

Citizens who vote for a candidate other than one of the two most likely to win waste their votes.

We cannot capture this idea in a model in which there is perfect information. To see why, suppose that several candidates offer different, fixed policies and there is a large number of citizens who may vote costlessly for any candidate. Then as we saw in Section 2.8 there are many equilibria; for example, all citizens could vote for any one of the candidates, even the one who is the favorite of the smallest number of citizens.

If we introduce some uncertainty, however, we can capture the idea, as Cox (1987a) and Palfrey (1989) cleverly demonstrate. (Palfrey's work builds on that of Ledyard (1981, 1984) in its formulation of the voting model and on that of Cox (1987a) in the specific application; Cox (1993) extends the result to voting systems more general than plurality rule.) Suppose that there is a large number of citizens, each of whom knows her own preferences, but not those of any other citizen. Given the finite set of candidates, there is a finite number of possible preference relations. The citizens are *a priori* identical; each citizen's preferences are drawn from the same distribution, known to all citizens. Restrict attention to symmetric equilibria, in which the candidate for whom a citizen votes depends only on the citizen's type.

Consider the candidate for whom citizen i should vote, given the behavior of all other citizens. Order the candidates so that, given the behavior of all other citizens, candidate 1's expected vote total is highest, candidate 2's is next highest, and so on. When the number of citizens is large it is very likely that candidate 1 will win the election, but so long as there is some uncertainty there is some chance that one of the other candidates will do so; candidate 2 is the next most likely to be the winner. Note that citizen i's vote makes a difference to the outcome only if it is pivotal—i.e. only if in its absence the winner would receive at most one more vote than the second-place candidate. Now, the point is that in a large electorate the probability that candidate j for $j \geq 3$ is one of the top two vote-getters is very small compared with the probability that either candidate 1 or candidate 2 is. Thus with very high probability citizen i's vote makes a difference only if it is cast for either candidate 1 or candidate 2. It follows that the optimal action for citizen i is to vote for whomever of candidates 1 and 2 she prefers. Most likely candidate 1 will be the winner by a margin of two or more votes, so that it makes no difference whom citizen i votes for. But there is some chance that, in the absence of citizen i's vote, candidate 2 either ties for first place with candidate 1 or obtains one less vote than candidate 1, so that if citizen i prefers candidate 2 she is better off voting for her. At the same time there is no point in voting for any other candidate even if neither candidate 1 nor candidate 2 is citizen i's favorite, since the probability that some other candidate will win is very small (smaller, the larger the electorate) compared with the probability that one of candidates 1 and 2 will win.

The conclusion is that in any equilibrium in which the expected number of votes received by the third-ranked candidate is less than the expected number received by the either of the first two candidates, no candidates except the first two receive any votes. That is, there are only at most two active candidates.

To rule out equilibria in which there is only *one* active candidate assume that (a) voters do not use dominated strategies and (b) for every candidate j there are some voters for whom j is the least preferred candidate. Then each candidate is some citizen's least preferred candidate, and hence it is a dominated strategy for the citizen to vote for her.

The model thus neatly formalizes the idea that votes for third parties are wasted. In doing so it points to a limitation of the idea: the game has equilibria in which many candidates tie for second highest expected number of votes and these and the first-ranked candidate all have a positive probability of winning, and also equilibria in which more than two candidates tie for first place. Further, in a model in which the candidates can choose their positions—i.e. in the full Hotelling–Ledyard game, not just in a voting subgame—such situations may arise endogenously. (Indeed, if there is perfect information and the candidates compete for votes as Feddersen *et al.* (1990) assume then in any equilibrium all the candidates choose the same position and receive the same number of votes (Proposition 5).)

4.2 Strategic Voting Under Perfect Information

Another explanation for the existence of only two parties is due to Feddersen (1992) (who builds upon the model of Feddersen *et al.* (1990)). The main idea in his model is the following.

> If voting is costly and information is good then the election will be close: all candidates with a positive probability of winning are very likely to obtain the same number of votes, so that every vote is very likely to be pivotal. Hence for each citizen there can be no more than one candidate who is preferred to the lottery over all the winning candidates.

As argued in Section 2.9.2 this idea leads to the conclusion that if the citizens' payoff functions are concave then in any equilibrium at most two positions are occupied. In the model of Feddersen *et al.* (1990) we can further conclude that there is no equilibrium in which exactly two positions are occupied. Feddersen (1992), however, provides a model in which two-position equilibria survive and the one-position equilibria that Feddersen *et al.* find do not. In his model there is perfect information and the only players are the citizens, who simultaneously vote, at a cost, for positions (see Section 3.1.3). The absence of strategic parties eliminates the incentive for the positions that receive votes to converge, with the consequence that there are equilibria in which just two (separated) positions receive votes. Further, if the preferences of the citizens are sufficiently diverse then there is no equilibrium in which just one position, say x, receives votes. The argument is as follows. In such an equilibrium at most one citizen votes. (If the outcome is that the candidate is elected even if no one votes, then no one indeed votes; if in the event that no one votes there is an outcome that is worse for at least one citizen than the candidate being elected then one citizen votes.) But now any citizen for whom x is not the most desirable position can vote for *her* favorite position, say y, leading to the outcome in which x and y each occur with probability $\frac{1}{2}$, which, so long as she prefers y to x by a wide enough margin, is preferable. Formally, the result is the following.

Proposition 7. *Consider the strategic game in which*

- *the set of players is I (the set of citizens)*
- *the set of actions of each citizen is $X \cup \{Abstain\}$ (where $X \subseteq \mathbb{R}$ is the set of possible positions), the action $x_i \in X$ of citizen i being interpreted as a vote for position x_i*
- *the payoff of each player i to a profile x of actions is given in (9), where $u_i \colon X \to \mathbb{R}$ is concave and $c > 0$.*

In any Nash equilibrium of this game at most two positions receive a positive number of votes. Further, if for every position $x \in X$ there is a citizen $i \in I$ and a position $y \in X$ such that $u_i(y) > u_i(x) + 2c$ then there is no equilibrium in which only one position receives a positive number of votes.

Outline of proof.

Step 1. In any equilibrium all positions that receive votes obtain the same number of votes. (If not, then one certainly loses, and any citizen who votes for it is better off abstaining, given the positive cost of voting.)

Step 2. In any equilibrium at most two positions receive votes. (If more than two positions do so then, given the concavity of the citizens' payoff functions, any citizen who votes for an extreme position can vote for a different position, cause that to be the outright winner, and increase her expected payoff.)

Step 3. In any equilibrium in which a single position receives all the votes exactly one citizen votes. (If more than one votes then any of them can switch to abstaining without affecting the outcome.)

Step 4. If the condition in the last sentence of the result is satisfied then there is no equilibrium in which a single position receives votes. (By Step 3 the only possibility is that the position, say x, receives a single vote. But then the citizen i for whom $u_i(y) > u_i(x) + 2c$ can vote for y and obtain the payoff $\frac{1}{2}(u_i(y) + u_i(x)) - c > u_i(x)$.) □

Step 2 of this proof uses the concavity of the payoff functions, and this assumption cannot be dispensed with entirely. Suppose, for example, that there are k candidates, and that for each candidate j the fraction $1/k$ of the citizens ranks j first and is indifferent between all the other candidates. Then there is an equilibrium in which all citizens vote sincerely, and all k candidates obtain the same number of votes. However, in the case covered by the proposition, in which the policy space is one-dimensional, the assumption of concavity can be relaxed considerably. If the policy space is higher-dimensional, on the other hand, the assumption of concavity is not enough; Feddersen (1992) obtains a result by assuming that the citizens' payoff functions are quadratic and that their ideal points are spread with sufficient uniformity over the policy space.

The result shows how powerful strategic voting can be in reducing the size of the set of equilibria in the presence of perfect information. It is not clear to what extent the result survives if voters are imperfectly informed about each others' characteristics. If it is not certain (as here) that all the winning positions will tie then it seems that there can be a configuration of votes for three or more candidates with the property that no citizen wants to switch her vote. However, combined with the arguments of Palfrey in the previous section the indication is that the forces leading to a two-position equilibrium are strong.

As discussed in Section 3.1.3, the absence of strategic candidates in Feddersen's model greatly reduces the incentive for convergence of the parties' positions, an incentive that is partly restored by Feddersen (1993), who uses the solution concept of coalition-proof Nash equilibrium (rather than Nash equilibrium). However, as argued earlier, even in this case it is not clear that the incentive for the candidates to separate is not an artifact of the structure of the model, in which all decisions are made simultaneously; the results may change if the parties take into account the reactions of voters when considering the positions to take.

4.3 Strategic Positioning With an Endogenous Number of Candidates

The ideas in the previous two sections appeal to voting behavior as the main factor limiting the number of parties under plurality rule. By contrast, the ideas in this section concern the role of the entry of new parties in limiting the number of active parties. The first idea is the following.

A small number of parties can choose positions with the property that any subsequent entrant loses.

This idea is formalized to a limited extent by Palfrey (1984). In his model (see Section 3.3) two vote-maximizing candidates simultaneously choose positions, then a third does so. In a subgame perfect equilibrium the third party enters, but certainly loses. This result is limited by the fact that there are only three potential candidates; further, as argued in Section 2.9.1, the assumption that candidates are vote-maximizers is significant, and lacks appeal. Palfrey points out (p. 154) that if n candidates choose simultaneously and there is a single follower then there is an equilibrium in which all $n+1$ candidates enter (see also Weber (1992a)); what happens when there is more than one follower is unclear.

In an alternative model that I have explored (Osborne (1993)), each of three potential candidates may enter whenever she wishes and candidates prefer to stay out of the competition than to enter and lose; in any subgame perfect equilibrium there is a *single* entrant (see Section 3.3). The single candidate forestalls further entry since an additional entrant makes it possible for another candidate to enter and win outright.

These two results suggest that the threat of future entry can indeed limit the number of candidates. However, the analyses are both limited; how the results fare when there are more than a small number of candidates or when information is imperfect is not clear.

A different idea is the following.

> If candidates care about the position of the winner and there are three or more candidates then at least one of the extreme candidates can withdraw, giving her votes to the next most extreme candidate and causing that candidate to win outright—an outcome that is better for her than that in which all candidates stay in the competition.

This idea is related to that in Section 4.2; one difference is that it rests on strategizing by candidates contemplating entry rather than citizens contemplating for whom to vote. It is formalized by Osborne and Slivinski (1993) in a model in which the players are the citizens; each citizen chooses whether to become a candidate, in which case she is restricted to offer her ideal position, or to stay out of the competition. Entry costs $c > 0$, while winning confers a benefit $b \geq 0$ (the "spoils of office") in addition to leading to a policy outcome that is desirable for the winner. In order to isolate the effect of citizens' entry and exit decisions on the outcome from the effects of strategic voting, we assume that voting is sincere. (We do not deny that a model in which citizens act rationally may best capture voting behavior. However, the environment in which voting takes place in the world is complex: there is imperfect information, and elections are held repeatedly, for example. In such an environment a citizen may be motivated to vote for a position that is sure to lose in the current election, in order to signal her support for that position. Consequently we argue that the *naïve* model of strategic voting *may* explain behavior no better than the model of sincere voting.)

The nature of the equilibria depend on the cost c and benefit b of running as a candidate. If b is small enough relative to c then in every equilibrium of the game there is a single candidate; for some range of larger values of b, in every equilibrium there are precisely two candidates, and for b even larger there are equilibria with more than two candidates. Since we observe many elections in which the number of candidates is not two, the model thus has an advantage over those in Sections 4.1 and 4.2, which do not yield conditions that are so directly related to observable variables under which multi-candidate equilibria exist. One interesting result is that in an equilibrium one of the candidates may *lose*: she may enter in order to change the identity of the winner to one whose position she prefers (did such a consideration partly motivate Perot in the U.S. Presidential election of 1992?). These results points to a limitation of the idea that is highlighted above: if holding office itself confers benefits that are large enough then extremists who have some chance of winning may not want to withdraw, and multi-candidate equilibria are possible.

The model is related to that of Feddersen (1993). In both cases parties emerge endogenously; in Feddersen's model a party is identified with the citizens who vote for a position, while in this model a party is identified with a single citizen. In Feddersen's model, each citizen decides whom to vote for, while in this model each citizen decides whether to stand as a candidate; to compare the two models we can think of the action of standing as a candidate in terms of the voting behavior that it implies. In Feddersen's model any coalition of voters may consider deviating. By contrast, in our model the implied deviations by voters are of only two types: the supporters of one position may *en masse* switch to the next nearest position (i.e. a citizen may withdraw as a candidate), or all the citizens for whom some new position is the most desirable one can form a party with that position (i.e. a citizen may become a candidate). On the one hand our model thus eliminates a strategic action that seems problematic in Feddersen's model—a move in a party's position that is assumed not to affect the support for the rival party; on the other hand we eliminate a strategic action that may be significant—we do not allow a party to consider changing its position at all. It thus appears that additional modeling could further improve our understanding of the incentives for candidate entry.

5 Concluding comments

The insight afforded by Hotelling's model cannot be underestimated. At the same time, most of the ideas designed to explain the stylized facts of political competition rely on features that are absent from his model: citizens who act strategically in their voting behavior, candidates who care about the policy of the winner of the election, imperfect information by candidates about the citizens and by the citizens about each other and about the candidates, possible entry by new candidates, and parties that are formed and run endogenously by the citizens. It seems likely that future work, even if it is rooted in Hotelling's spatial framework, will continue to incorporate features like these, with the result that its implications differ significantly from those of Hotelling's model.

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