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Intraday Trading Patterns: The Role of Timing

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Abstract

In a dynamic model of financial market trading multiple heterogeneously informed traders choose when to place orders. Better informed traders trade immediately, worse informed delay — even though they expect the public expectation to move against them. This behavior causes distinct intra-day patterns with decreasing (L-shaped) spreads and increasing (reverse L-shaped) volume and probability of informed trading (PIN). Competition increases market participation and causes more pronounced spread and less pronounced volume patterns. Systematic improvements in information increase spreads and volume. Very short-lived private information generates L- or reverse J-shaped volume patterns, which are further enhanced by competition.

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One persistent empirical finding in analyses of stock market trading data is that volume and spreads display intra-day patterns. These patterns differ across markets and across the analyzed time spans, but, most commonly, the spread declines and volume increases toward the end of the trading day. For instance, NYSE historically displayed U- or reverse J-shaped spreads and volume (Jain and Joh (1988), Brock and Kleidon (1992), McInish and Wood (1992), Lee, Mucklow, and Ready (1993), or Brooks, Hinich, and Patterson (2003)), but recent evidence (Serednyakov (2005)) suggests L-shaped spreads after decimalization; NASDAQ has L-shaped spreads and U-shaped volume (Chan, Christie, and Schultz (1995)); London Stock Exchange has L-shaped spreads and reverse L-shaped volume (Kleidon and Werner (1996) or Cai, Hudson, and Keasey (2004)).[1]

Persistent patterns in transaction costs have long puzzled researchers — why trade at high transaction costs mid-day when on average costs are lower at the end of the trading day? The existing literature, discussed below, provides several explanations, such as periodic variations in uninformed trading or short-lived information advantages. We identify a new channel. In our model, intra-day patterns arise endogenously through the dynamic behavior of heterogeneously informed traders. We further contribute to the literature by deriving novel predictions on how key market features such as competition among traders, transparency through information release policies, and the structure of private information affect these patterns.

The theoretical model underlying our analysis is in the tradition of Glosten and Milgrom (1985). Liquidity is supplied by a competitive, uninformed, and risk neutral market maker. Traders either place orders for reasons outside the model (e.g., to rebalance their portfolio), or they have private information about the security's fundamental value. In contrast to Glosten and Milgrom, we allow the latter traders to choose the time of their trade and we admit an uncertain number of traders.

[1]There are many other examples. For instance, the Taiwan and the Singapore Stock exchanges have L-shaped spreads and reverse L-shaped volume or number of transactions (Lee, Fok, and Liu (2001) for Taiwan, Ding and Lau (2001) for Singapore).

As a first step in our analysis, we verify that our model is consistent with the aforementioned, common patterns in observables. At each point in time the market maker sets a bid price and an ask price. Traders with private information expect prices to move against them because they believe that their peers likely possess similarly favourable or unfavourable information.[2] This effect is stronger, the more confident traders are about their information. Consequently, the best informed traders act early, causing a wide bid-ask spread. Less well informed trader are unwilling to accept this wide spread and, further, they do not expect prices to move by much. They are thus happy to delay, causing a natural separation and a declining (L-shaped) spread pattern across time.

The bid-ask-spread is affected, loosely, by the average information quality of active traders and by the probability that an informed trader is active. In the degenerate case with only one trader, spreads must coincide across time. As a trader with better information would move early, on average the informed trader transaction rate must be increasing across time, leading to a reverse L-shaped volume pattern. This intuition extends to situations with more than one trader.

Our first main result determines the impact of competition among traders. As the expected number of traders rises, traders are increasingly concerned that the price moves against them and more traders act early to capitalize on their information. This mutes the reverse L-shaped volume pattern and leads to a more pronounced L-shaped spread pattern. Notably, the steeper decline in spreads leads to an overall increase in market participation. Competition for information rents thus does not deter but attracts market entry and allows traders to benefit from weaker information.

The second main result concerns the impact of a possible public signal that renders private information obsolete after the first period of trading. As traders believe that their information may be very short lived, they feel compelled to act sooner. We then predict that as the threat of an information release increases, the L-shaped spread pattern becomes

[2] Transaction prices in our model are a martingale and information is independent, conditional on the security's fundamental value. This causes private information to be unconditionally correlated.

more pronounced and the reserve L-shaped volume pattern becomes muted. When traders perceive their private information to be sufficiently short-lived, volume declines over time.

Our third empirical prediction concerns the impact of systematic improvements of private information. Such an improvement can occur, for instance, when a company adopts or a regulator imposes a new disclosure policy that fosters transparency.[3] Our model predicts that, ceteris paribus, stocks of companies with such new policies exhibit higher total volume and higher spreads. Further, L-shaped spread patterns are more pronounced and reverse L-shaped volume patterns are less pronounced.

Our fourth result relates volume and the probability of informed trading (PIN), which is a widely used empirical measure of adverse selection costs (Easley, Kiefer, O'Hara, and Paperman (1996)). PIN assesses the degree of activity by informed traders, and our model predicts that PIN follows the pattern of volume. We thus predict that spreads and PIN, two main measures of adverse selection costs, will display opposite patterns when volume and spreads display opposite patterns. The intuition behind the discrepancy is that PIN captures the chance of encountering an informed trader, whereas the spread additionally accounts for the level of the informational advantage of such a trader.

The literature has developed several explanations for persistent variations in observable variables. Admati and Pfleiderer (1988) analyze a setting in the tradition of Kyle (1985) and attribute periods of concentrated trading to the timing decisions of discretionary liquidity traders. Informed traders do not time their actions as their information is viable for only one period. The period with highest activity is determined by exogenous parameters and thus, in principle, their model admits any pattern. Foster and Viswanathan (1990) analyze a single informed trader model and show that *inter*-day variations in volume and transaction costs arise when there is release of public information.[4] We complement their

[3]Related to this are many examples of incremental or even dramatic improvements in economy wide information quality, such as the advent of new data sources or new computing tools that allow faster processing of data. Our model then delivers testable predictions for event studies of such changes.

[4]Other effects caused by the timing decision of a *single* informed trader have been analyzed in, for instance, Back and Baruch (2007) (order splitting), Chakraborty and Yilmaz (2004) (price manipulation), and Smith (2000) ((no-) timing in absence of bid-ask spreads).

work and offer predictions on the impact of competition and information structures.

Finally, employing an inventory based trading model, Brock and Kleidon (1992) show that U-shaped volume can be caused by demand shocks that traders experience during periods of market closure. The monopolistic market maker then exploits this pattern and charges U-shaped spreads. Our analysis complements this line of work by studying *competitive* liquidity provision in a setting with asymmetric information.

Overview. Section 1 outlines the model, Section 2 derives the equilibrium. Section 3 studies the effect of an increase in competition between traders on market participation. Section 4 discusses the patterns of spreads, volume and PIN. Section 5 analyzes the impact of a possible release of public information. Section 6 determines the effect of systematic improvements in private information. Section 7 discusses the results. Appendix A expands on the information structure. Appendix B complements the main text by providing the proofs and further details of the analysis. Table 1 summarizes the results.

1 The Model

1.1 Overview of the Market Structure

We formulate a stylized model of security trading, in which traders trade single blocks of a risky asset with a competitive market maker. Our model builds on Glosten and Milgrom (1985) (hereafter, GM) but we assume that more than one trader may arrive at the same time and we allow traders to time their transactions.

There are two trading periods. At the beginning of period 1, before trading commences, the fundamental value of the security is realized (but not revealed) and some investors receive private information about this value. These traders are rational and trade to maximize their expected profits. If not informed, a trader may experience a liquidity shock forcing him to buy or sell.[5]

Trading is organized by a competitive market maker who posts 'bid'-prices, at which

[5]Throughout the paper we will refer to market makers as female and investors as male.

she is willing to buy the security, and offer-prices ('ask'), at which she is willing to sell. These quotes are 'good for' the size of a single block, and the market maker expects to break even when the quote is hit by a single order. If multiple orders arrive simultaneously, they exceed the minimum fill size for posted orders, and new quotes are negotiated.

The value of the security is revealed at the end of period 2, after the market closes. Traders who hold the asset obtain its cash value and consume. Short positions are filled at the fundamental value.

1.2 Model Details

Security: There is a single risky asset with a liquidation value V from a set of two potential values $\mathbb{V} = \{0, 1\}$. The two values are equally likely.

Market maker: The market maker is risk-neutral and competitive. She does not have private information and sets prices to break even, conditional on the public information.

Traders: With probability α there are two traders, with probability $1 - \alpha$ there is only one trader.[6] A trader is equipped with private information about the value of the security with probability $\mu \in (0, 1)$. The informed investors are risk neutral and rational.

If not informed (probability $1 - \mu$), a trader may experience a liquidity shock.[7] To simplify the exposition, we assume that this liquidity shock occurs with probability 1, that it is equally likely to occur in either of the two trading periods, and that it forces the trader to buy or sell with equal probabilities.

Informed traders' information: We follow most of the GM sequential trading literature and assume that traders receive a binary signal about the security's fundamental value V. These signals are private, and they are independently distributed, conditional

[6]Most results in this paper numerically extend to a setting with an arbitrary number of traders determined by a Poisson process; the qualitative insights remain unaffected.

[7]Assuming the presence of traders who trade for exogenous reasons ('noise') is common practice in the asymmetric information literature to prevent "no-trade" as in Milgrom and Stokey (1982).

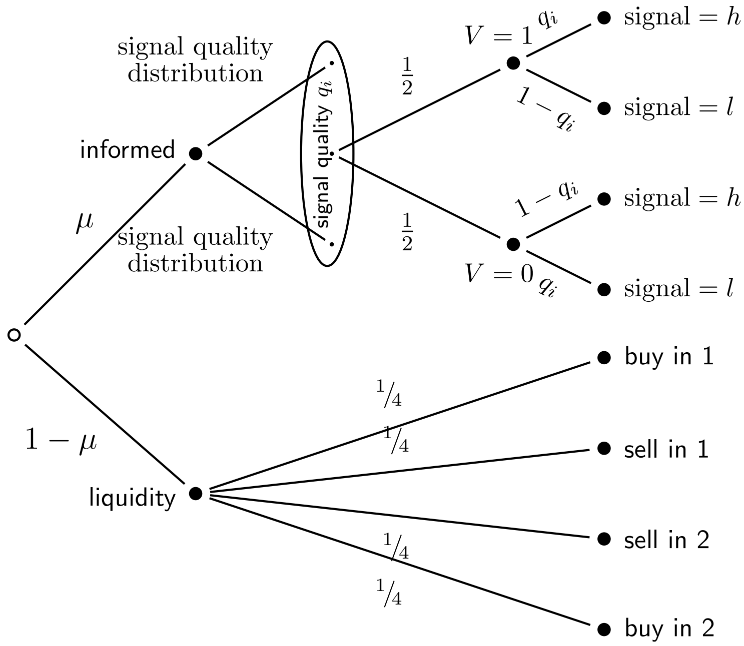


Figure 1: **Illustration of signals and noise.** This figure illustrates the structure of information and noise trading. First, it is determined whether a trader is informed (probability μ) or uninformed (probability $1-\mu$). If informed, the trader obtains a signal quality q_i. Next, he receives the "correct" signal (h when $V=1$ and l when $V=0$) with probability q_i and the "wrong" signal with probability $1-q_i$. (The draw of V is identical for all traders.) If the trader is not informed, he experiences a liquidity shock in Periods 1 and 2 with equal probabilities.

on V. Specifically, trader i is told "with chance q_i, the value is High/Low (h/l)" where

pr(signal|true value)	$V=0$	$V=1$
signal $= l$	q_i	$1-q_i$
signal $= h$	$1-q_i$	q_i

This q_i is the *signal quality.* In contrast to most of the GM literature, we assume that these signals come in a *continuum* of qualities, and that q_i is trader i's private information. The distribution of qualities is independent of the security's true value and can be understood as reflecting, for instance, the distribution of traders' talents to analyze securities.

In what follows, we will combine the binary signal (h or l) and its quality on $[1/2, 1]$ in a single variable on $[0,1]$, namely, the trader's private belief that the security's fundamental value is high ($V = 1$). This belief is the trader's posterior on $V = 1$ after he learns his quality and sees his private signal but *before* he observes the public history. A trader's behavior given his private signal and its quality can then be equivalently described in terms of the trader's private belief. This approach allows us to characterize the equilibrium in terms of a continuous scalar variable (as opposed to a vector of traders' private information) and thus simplifies the exposition.

Trader i's belief is obtained by Bayes Rule and coincides with the signal quality if the signal is h, $\pi_i = \mathsf{pr}(V = 1|h) = q_i/(q_i + (1 - q_i)) = q_i$. Likewise, $\pi_i = 1 - q_i$ if the signal is l. Appendix A fleshes out how the distributions of beliefs are obtained from the underlying distribution of qualities and provides several examples. Figure 1 illustrates the structure of information and liquidity trading.

Public and private information: The number of traders in the market is not revealed, except possibly through submitted orders. The identity of an investor (informed or liquidity), his signal and his signal quality are his private information. Past trades and transaction prices are public information. We will use H to summarize the public information about everything that occurred in period 1.

Trading protocol: There are two trading periods, $t = 1, 2$. As in GM, each trader can post at most one market order to either buy or sell one block of the security at prices determined by the market maker. An informed trader in our setting can additionally choose the period to submit his order in.[8] Informed traders choose the period and the direction of their trade (or to abstain from trading) to maximize their expected profits.

The market maker's bid and ask quotes take into account the information that single sell and buy orders, respectively, would reveal. When two orders arrive at the same time, the market maker may quote a new price to each of the traders. When faced with an updated quote, a trader is allowed to reject it. If he does, then this trader leaves the market and makes zero profits.[9] Figure 2 illustrates traders' choices.

[8]Our results are qualitatively robust to allowing a fraction of liquidity traders to time their trades.

[9]Since trading is non-anonymous, a trader who rejects the new price in period 1 and returns later

The assumption that traders may reject the updated quotes reflects the reality that block orders are commonly traded off the exchange at mutually agreed upon prices, and it is thus unlikely that a block trader faces uncertainty about the execution price. We would like to emphasize, however, that this assumption is not crucial to our results. We will show that, in equilibrium, traders will always accept the updated quotes. This implies, in particular, that equilibrium transaction prices in our model would also constitute an equilibrium in a dealer market setting in the tradition of Kyle (1985), where traders first submit market orders and the market maker then sets the transaction price upon observing the order flow. As one of this paper's goals is to describe the effect of timing and competition on the dynamics of the bid-ask spread, we have chosen the formulation that admits such a description over the dealer market alternative.

1.3 Trading Equilibrium

Quotes: At the beginning of period t the market maker posts a bid- and an ask-price; the bid-price bid^t is the price at which she buys exactly one unit of the security, the ask-price ask^t is the price at which she sells exactly one unit. With zero expected profits, these quotes coincide with the market maker's conditional expectation of the fundamental

$$\mathsf{ask}^t = \mathsf{E}^{\mathsf{M}}[V|\text{buy at } \mathsf{ask}^t,\ \text{public info at } t], \quad \mathsf{bid}^t = \mathsf{E}^{\mathsf{M}}[V|\text{sale at } \mathsf{bid}^t,\ \text{public info at } t].$$

If only a single order arrives, then it clears at the quoted price. If there are two orders, then the market maker updates the quotes based on the information revealed by the orders. Buyers and sellers are then quoted new prices

$$\begin{aligned}\mathsf{ask}^t_{\mathsf{new}} &= \mathsf{E}^{\mathsf{M}}[V|\text{other order and buy at } \mathsf{ask}^t,\ \text{public info at } t],\\ \mathsf{bid}^t_{\mathsf{new}} &= \mathsf{E}^{\mathsf{M}}[V|\text{other order and sale at } \mathsf{bid}^t,\ \text{public info at } t].\end{aligned}$$

is identified and thus cannot profit from doing so. Further, we will show that a trader who initially submitted a buy order will never want to sell upon hearing the new quotes.

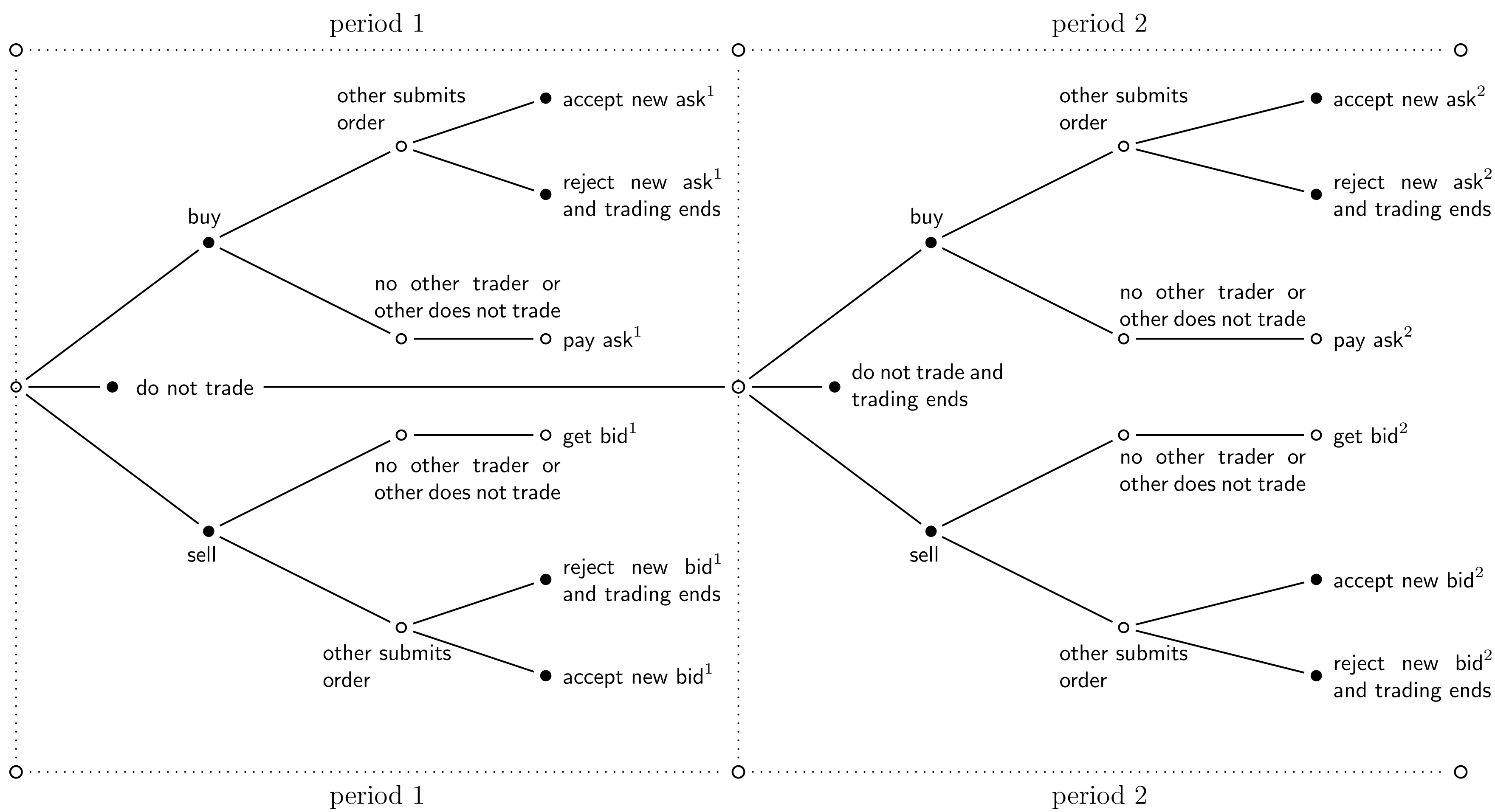


Figure 2: **Decision Possibilities and Possible Outcomes from the Perspective of one Trader.** In each period, when (a) there is no other trader, (b) there is another trader who does not trade, or (c) the other trader has already traded (applies only to period 2), the trader has to pay the quoted ask price, ask^t, or will obtain the quoted bid price, bid^t. If there is another trader who submits an order, then the market maker will update her expectation and quote a new price. The trader then has the choice to either accept or reject this quote. Trading ends for a trader if he rejects a quote.

The informed investor's optimal choice: An informed investor enters the market in period 1. He can submit a buy or a sell order in this period, or he can delay his decision until period 2. A trader who does not submit his order in either of the two trading periods makes zero profits. As discussed earlier, if two orders are submitted at the same time, each trader will be quoted an updated price and can then choose to either proceed with his order or to abstain from trading.

Equilibrium concept: We will restrict attention to monotone threshold decision rules. Namely, we will seek an equilibrium where traders with sufficiently encouraging (discouraging) signals buy (sell) in period 1 and those with information of worse quality choose to delay their decision until period 2. Further, we will focus on a symmetric equilibrium, where quality thresholds are independent of the trader's identity and, absent transactions, buyers and sellers require signals of (or above) the same quality to trade. If an equilibrium is not unique, we select the volume maximizing equilibrium.

More formally, we will look for an equilibrium such that a trader buys in period 1 if his private belief $\pi_i \in [\pi_b^1, 1]$ and sells in period 1 if $\pi_i \in [0, \pi_s^1]$; that he buys in period 2 if his private belief $\pi_i \in [\pi_b^2(H), \pi_b^1)$ and sells if $\pi_i \in (\pi_s^1, \pi_s^2(H)]$; and that he abstains from trading otherwise (where period 2 thresholds may depend on the period 1 history H). Symmetry with respect to buying and selling implies that $\pi_b^1 = 1 - \pi_s^1$ and that, conditional on $H =$ 'no transaction at $t = 1$', $\pi_b^2(H) = 1 - \pi_s^2(H)$. The volume maximizing equilibrium has the lowest π_b^t and the highest π_s^t.

2 Equilibrium Analysis

We proceed in three steps. We first outline general properties of the equilibrium. Second, we describe the equilibrium in period 2, and third we discuss the equilibrium in period 1. Figure 3 summarizes the equilibrium decisions.

2.1 General Properties

Threshold decision rules, together with conditional independence of the traders' signals, imply that the probability of order history o^t, conditional on the security's fundamental value being $V = v$ and the trader's private information, is independent of the trader's private belief, $\mathsf{pr}(o^t|V = v, \pi) = \mathsf{pr}(o^t|V = v)$. The trader's expectation of the fundamental can then be written as

$$\mathsf{E}[V|\pi, o^t] = \frac{\pi \mathsf{pr}(o^t|V = 1)}{\pi \mathsf{pr}(o^t|V = 1) + (1 - \pi)\mathsf{pr}(o^t|V = 0)}, \tag{1}$$

and it is increasing in the private belief, conditional on any order history.

In a symmetric equilibrium, the expectation of a trader with private belief $\pi = 1/2$ coincides with that of the market maker, $\mathsf{E}[V|o^t, \pi = 1/2] = \mathsf{E}^{\mathsf{M}}[V|o^t]$, for any order history. As the market maker's quotes account for the information revealed by the order, the ask quote will be above and the bid quote will be below the market maker's expectation,

$$\mathsf{ask}(o^t) = \mathsf{E}^{\mathsf{M}}[V|o^t, \text{buy}] \geq \mathsf{E}^{\mathsf{M}}[V|o^t] \geq \mathsf{E}^{\mathsf{M}}[V|o^t, \text{sale}] = \mathsf{bid}(o^t),$$

with equality on either side only if *all* traders buy or sell. The above discussion implies

Lemma 1 (Separation of Trading Decisions) *A trader with private belief $\pi > 1/2$ would never sell, a trader with private belief $\pi < 1/2$ would never buy, and a trader with private belief $\pi = 1/2$ would never trade.*

In what follows, we will discuss the decision of a trader with private belief $\pi > 1/2$ in more detail; the discussion for the case of $\pi < 1/2$ is analogous. Employing Lemma 1, at the beginning of period 1 a trader with private belief $\pi > 1/2$ chooses between submitting a buy order and delaying his trading decision until period 2. He will submit a buy order in period 1 only if he expects to (i) make non-negative expected trading profits and (ii) these profits exceed those that he expects to make by delaying until period 2. This trader thus needs to first forecast his period 2 trading decision and profits, conditional on all possible period 1 transaction histories (a buy, a sale, or a no trade).

2.2 The Trading Decision in Period 2

We first fix the period 1 trading thresholds, $\pi_b^1 = \pi^1$ and $\pi_s^1 = 1 - \pi^1$, and find the period 2 trading thresholds for each of the period 1 trading histories. An informed investor with a private belief $\pi > 1/2$ submits a buy order in period 2 if, conditional on his information and the period 1 transaction history, the expected ask price is at or below his expectation of the security's fundamental value. He abstains from trading otherwise.

In a monotone equilibrium, this trader will submit a buy order after period 1 history H if his private belief π is at or above the belief of the marginal buyer who is exactly indifferent between submitting a buy order and abstaining from trade, $\pi_b^2(H, \pi^1)$. To characterize trader π's period 2 decision, it thus suffices to find the threshold belief $\pi_b^2(H, \pi^1)$.

Suppose first that there is a buy in period 1. Conditional on a buy order in period 2, the market maker will know that (*i*) there were two traders in the market, (*ii*) the period 1 buy order came either from a liquidity trader (and is thus uninformative) or from an informed trader with private belief between π^1 and 1, and (*iii*) the period 2 buy order came either from a liquidity trader or from an informed trader with private belief between $\pi_b^2(\text{'buy at } t = 1\text{'}, \pi^1)$ and π^1.

Since traders' decisions are independent, conditional on the fundamental value, a trader in period 2 and the market maker will derive the same information about the fundamental from (*i*) and (*ii*). For the marginal buyer's expectation to equal that of the market maker (which is the ask price), this buyer must derive the same information from his private signal as the market maker does from (*iii*). The latter point is the equilibrium condition. We show in Appendix B that this marginal buyer uniquely exists.

Consider now the case of no transaction in period 1. A trader who submits a buy order in period 2 knows that he may receive an updated quote from the market maker, and that he will be allowed to reject the new quote if it is 'too high'.[10] This option to reject has a non-negative value, and it is hypothetically possible for the marginal buyer to make negative expected profits at the posted ask price and positive profits if presented with an updated quote. Appendix B shows, however, that this cannot happen in equilibrium, and that the marginal buyer in period 2 will break even, conditional on any period 2 order history. Further, this marginal buyer is independent of the period 1 transaction history, $\pi_b^2(H, \pi^1) = \pi_b^2(\pi^1)$. Finally, the marginal seller is symmetric, $\pi_s^2(\pi^1) = 1 - \pi_b^2(\pi^1)$. In what follows, we drop the subscripts and use $\pi^2(\pi^1) = \pi_b^2(\pi^1)$ for the marginal buyer.

Lemma 2 (Marginal Traders in Period 2) *For any $\pi^1 \in (1/2, 1)$, there exist a unique $\pi^2 \in (1/2, \pi^1)$ such that any trader with private belief $\pi \in [\pi^2, \pi^1)$ buys in period 2, any trader with private belief $\pi \in (1 - \pi^1, \pi^2]$ sells in period 2, and any trader with private belief $\pi \in (1 - \pi^2, \pi^2)$ abstains from trading. Further, $\pi^2(\pi^1)$ increases in π^1.*

2.3 The Trading Decision in Period 1

Similarly to period 2, we can find a trader who is indifferent between buying in period 1 and abstaining forever if he is perceived to be the marginal buyer. Denote this buyer's belief by π^*. Lemma 2 showed, however, that if perceived to be the marginal buyer in period 1, trader π^* will make positive expected profits by delaying and submitting his buy order in period 2. Consequently, the marginal buyer in period 1, π^1, must be above π^*.

Trader π^1 will thus make strictly positive trading profits, conditional on any order history, and he will always accept an updated quote if presented with it. He solves

$$\mathsf{E}[V - \mathsf{ask}^1 | \pi^1, \text{I submit } \mathsf{B} \text{ at } t = 1\,] = \mathsf{E}\left[\mathsf{E}[V - \mathsf{ask}^2 | \pi^1, H, \text{I submit } \mathsf{B} \text{ at } t = 2\,] | \pi^1\right], \quad (2)$$

where trader π^1 conditions on himself being the marginal buyer in period 1, on trader $\pi^2(\pi^1)$ being the marginal buyer in period 2, and on symmetric marginal sellers. Using

[10] By Lemma 1, this trader will never choose to sell upon hearing the updated quote.

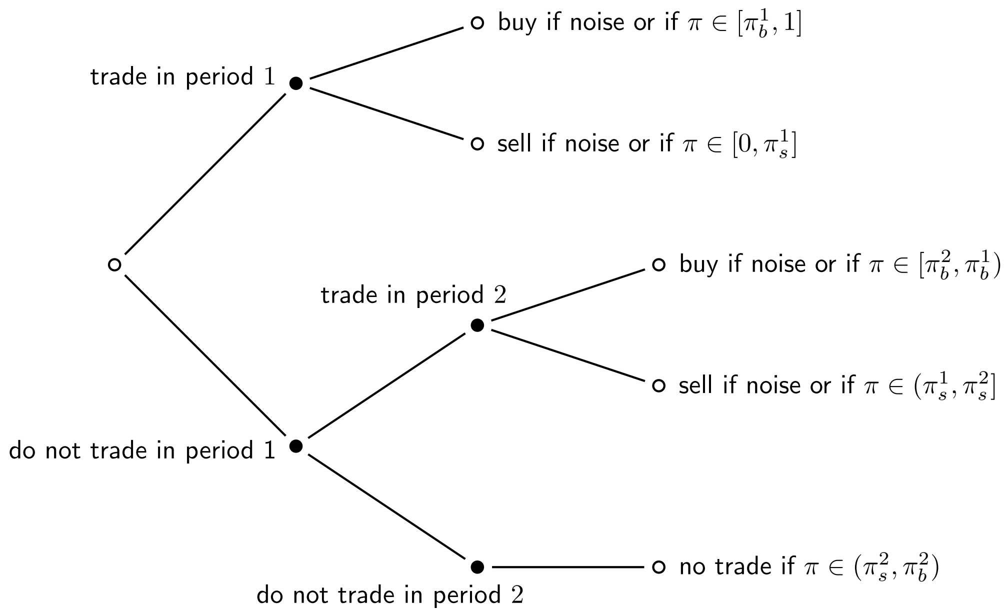


Figure 3: **Equilibrium Behavior.** This figure illustrates an informed trader's *equilibrium* choices. He acts in period 1 if his private belief is either low or high, and he delays otherwise; similarly in period 2.

the fact that updated quotes are always accepted, we can apply the Law of Iterated Expectations and rewrite equation (2) as a condition on the expected ask prices

$$\mathsf{E}[\mathsf{ask}^1|\pi^1, \text{I submit } \mathsf{B} \text{ at } t = 1\ ] = \mathsf{E}\left[\mathsf{E}[\mathsf{ask}^2|\pi^1, H, \text{I submit } \mathsf{B} \text{ at } t = 2\ ]|\pi^1\right]. \tag{3}$$

Theorem 1 (Existence of a Symmetric Equilibrium) *There exist π^1, π^2 with $1/2 < \pi^2 < \pi^1 < 1$ such that any trader with private belief $\pi \in [\pi^1, 1]$ buys in period 1, any trader with private belief $\pi \in [\pi^2, \pi^1)$ buys in period 2, and no trader with private belief $\pi < \pi^2$ buys. Selling decisions are symmetric.*

3 Competition and Market Participation

To better understand traders' incentives, consider first the case of $\alpha = 0$, when there is only one trader in the market. In a monotone equilibrium, this trader will buy in period 1 if his private belief π is at or above π^1, and he will buy in period 2 if his private belief π is at or above $\pi^2 > 1/2$ but below π^1. Being alone in the market, the trader can perfectly forecast the period 2 ask quote. Further, by the Law of Iterated Expectations, he cannot expect his expectation of the fundamental to change from period 1 to period 2. Consequently, conditional on submitting a buy order, this trader will always submit his order in period 1 if the period 1 ask price is lower than the period 2 ask price, $\mathsf{ask}^1 < \mathsf{ask}^2$, and vice versa. For this trader to trade in either period, depending on his belief, we must have $\mathsf{ask}^1 = \mathsf{ask}^2$, or in other words, the adverse selection costs must coincide across periods. Lemma 5 in Appendix B shows that this equality uniquely determines trader π^1.[11]

At first sight, it may seem counterintuitive that prices coincide across periods. Casual intuition suggests that the bid-ask spread should be wider in period 1 because informed traders there have higher quality information. This intuition does not recognize , however, that the size of the spread depends not only on the information that informed traders possess but also on the chance that an informed trader with the relevant information exists. For prices to coincide, the market maker must be more likely to encounter an informed trader in period 2 than in period 1. In the single trader equilibrium the information quality and informed trader scarcity effects exactly offset one another.[12]

The observation that the volume of informed trading is higher in period 2 is also the key insight to the results that we discuss in the next section.

Now suppose that a trader faces potential competition from another block trader, $\alpha > 0$. The trader's decision then depends on two factors: the (expected) spreads and the price impact of a competing order. The first key observation is that a trader with favourable information believes that another informed trader, should he be present, is

[11]Note that the situation with $\alpha = 0$ is degenerate in the sense that all informed types are indifferent between trading at any time. With competition, $\alpha > 0$, non-marginal traders strictly prefer to trade in one period or the other.

[12]This insight is in contrast to Smith (2000): under the information structure here, Smith would predict that a single trader *always* wants to trade early. The reason for the discrepancy is that in Smith a trader has no price impact and thus faces a zero bid-ask spread.

more likely to buy than to sell.[13] Thus traders expect the public expectation to move against them and they will only delay if the spread in period 2 is smaller than in period 1.

The second key observation is that the higher a trader's information quality is, the more he expects the public expectation to move. Thus for traders with high quality information, a lower period 2 spread is an insufficient incentive to delay and they choose to trade at the larger period 1 spread. Crucially, although traders with low quality information also expect the public expectation to increase, they expect the ask price in period 2 to be lower than the period 1 ask price.

We show that, compared to the no competition case, in equilibrium more traders act early. Further, and somewhat surprisingly, as competition increases, traders require lower quality information to submit orders and thus market participation increases.

Proposition 1 (Competition and Market Participation) *As the probability α that another trader is present increases, market participation increases in period 1 and in periods 1 and 2 combined: π^1 and π^2 decrease.*

4 Patterns in Observables

Empirically, observable variables display intra-day patterns. Spreads are L-shaped on most markets, examples are NASDAQ (Chan, Christie, and Schultz (1995)), the London Stock Exchange (Kleidon and Werner (1996) or Cai, Hudson, and Keasey (2004)), Taiwan (Lee, Fok, and Liu (2001)) and Singapore (Ding and Lau (2001)). Early evidence for NYSE suggested that spreads are reverse J-shaped (Jain and Joh (1988), Brock and Kleidon (1992), McInish and Wood (1992), Lee, Mucklow, and Ready (1993), or Brooks, Hinich, and Patterson (2003)) but there is recent evidence that the spread pattern has become L-shaped after decimalization (Serednyakov (2005)).

[13] Signals and actions are independent across traders, conditional on the true value, but unconditionally they are correlated.

The patterns of volume differ across markets.[14] On NYSE and NASDAQ volume is U- or reverse J-shaped. On the London Stock Exchange, volume is reverse L-shaped, with two small humps, one in the morning and the other in the early afternoon. Other world markets, for instance, the Taiwan and the Singapore Stock exchanges also have a reverse L-shaped volume or number of transactions.

We will argue now that our model generates patterns in observable variables that are consistent with the above empirical observations. Namely, spreads in our model decline from period 1 to period 2 and are thus arguably L-shaped. Volume patterns depend on how short-lived traders perceive their private information to be. If the information is perceived to last through the trading day, then volume will increase from period 1 to period 2 and is thus, arguably, reverse L-shaped. If, however, traders believe that their information is very short-lived (for instance, due to the potential release of public information), then volume will decrease from period 1 to period 2 and can be viewed as L- or reverse J-shaped. We will also discuss how the level of competition and changes in information quality affect the predicted volume and spread patterns.

In this section we will continue to assume that traders' private information lasts until the end of period 2. In Section 5, we will relax this assumption and investigate the effect of a possible release of public information after period 1.

4.1 Spread Patterns

The bid-ask spread is the difference between the ask price and the bid price,

$$\mathsf{spread}^t = \mathsf{ask}^t - \mathsf{bid}^t.$$

In models with asymmetric information the size of the spread is associated with the implied adverse selection costs. To facilitate the comparison, we compare the prices that are quoted at the beginning of period 1 with those that are quoted at the beginning of period 2 in absence of transactions in period 1.

[14]The references are the same as for spreads.

The dynamic behavior of spreads in our model is driven by the monotonicity of decision rules and the incentive compatibility of trading. As discussed in the last section, in presence of competition adverse selection costs in period 1 are higher than in period 2. Further, as competition increases, the spread widens in period 1 and it narrows in period 2.

Proposition 2 (Spreads) *For every level of competition $\alpha > 0$, $\mathsf{spread}^1 > \mathsf{spread}^2$. As α increases, spread^1 increases and spread^2 decreases.*

Proposition 2 implies, in particular, that the difference in spreads $\mathsf{spread}^1 - \mathsf{spread}^2$ increases as competition increases. In other words, stronger competition leads to a more pronounced L-shaped spread pattern.

4.2 Volume Patterns

We proxy volume by the probability that a given market participant trades, which is the probability of a buy plus the probability of a sale,

$$\mathsf{vol}^t = \mathsf{pr}(\text{a given trader buys at } t) + \mathsf{pr}(\text{a given trader sells at } t).$$

As the competition parameter α increases, there will be more traders and thus more transactions, ceteris paribus. Our measure is not contaminated by this direct effect of an increased number of traders but instead captures volume per capita.

In the last section we argued that volume must increase from period 1 to period 2 for the single trader case ($\alpha = 0$). Our numerical analysis shows that the same holds for any $\alpha > 0$. Further, by Proposition 1, as competition increases, more traders act in period 1 and overall. Our numerical results show that more traders act in period 1 relative to period 2 so that the reverse L-shaped pattern becomes less pronounced. Figure 4 illustrates the following observation.

Numerical Observation 1 (Volume) *For every level of competition $\alpha \geq 0$, $\mathsf{vol}^2 > \mathsf{vol}^1$. As α increases, $\mathsf{vol}^1 - \mathsf{vol}^2$ increases.*

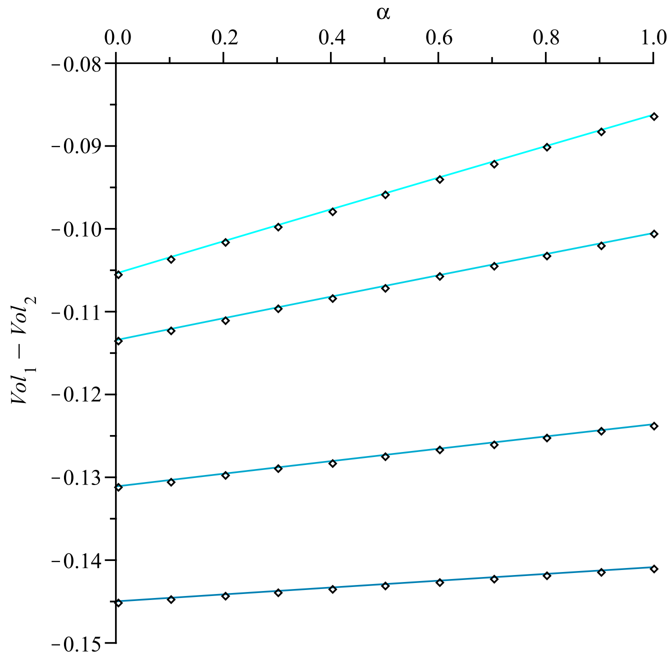


Figure 4: **Volume Patterns.** Competition parameter α is on the horizontal axis, $\mathsf{vol}^1 - \mathsf{vol}^2$ is on the vertical axis, the probability of an informed trader is set to $\mu = .4$. Each line corresponds to a parameter of the quadratic quality distribution outlined in Appendix A; the parameters are $\theta \in \{-6, 0, 6, 12\}$. While the volume difference increases in α, it remains negative, and volume in period 1 is lower than in period 2.

4.3 Patterns in the Probability of Informed Trading

Adverse selection costs in the context of Glosten-Milgrom sequential trading models are commonly measured by the "probability of informed trading" (PIN), introduced by Easley, Kiefer, O'Hara, and Paperman (1996). PIN is defined as the ratio of the probability of an informed transaction to the total probability of a transaction.[15]

$$\mathsf{PIN}^t = \frac{\mathsf{pr}(\text{a given trader acts in } t) - \mathsf{pr}(\text{a given trader is a noise trader})}{\mathsf{pr}(\text{a given trader acts in } t)} = \frac{\mathsf{vol}^t - 2\lambda}{\mathsf{vol}^t}. \quad (4)$$

We distinguish PIN for periods 1 and 2, where to facilitate the comparison, PIN in period 2 assumes no transactions in period 1. Relation (4) implies the following.

Proposition 3 (PIN) PIN^t *displays the same intra-day pattern as* vol^t*.*

[15]See Hasbrouck (2007), Ch. 6; the formula there has an additional parameter, which is 1 in our setting.

Thus, for instance, $\mathsf{PIN}^1 < \mathsf{PIN}^2$ when $\mathsf{vol}^1 < \mathsf{vol}^2$. Surprisingly, even though spread is larger in period 1, PIN is larger in period 2. In other words, these two measures of adverse selection costs disagree. The reason for the discrepancy is that PIN captures the chance of encountering an informed trader whereas the spread additionally accounts for the level of the informational advantage (or the average information quality) of an informed trader.

5 The Impact of a Public Information Release

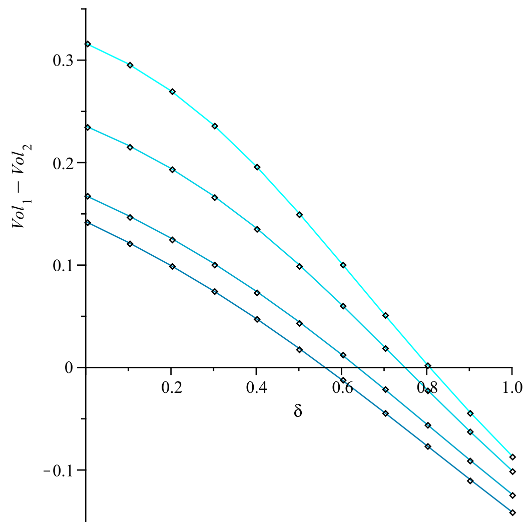
In the analysis presented thus far private information was assumed to be viable for two periods. We will now relax this assumption and discuss a situation where the private information may become obsolete after period 1. Private information may lose its value, for instance, because of a public announcement that perfectly reveals the fundamental value. Denoting the probability of such an announcement by $1-\delta$, $\delta \in [0,1]$, equation (2), which determines the marginal buyer π^1, becomes[16]

$$\mathsf{E}[V - \mathsf{ask}^1 | \pi^1, \text{I buy at } t=1] = \delta \cdot \mathsf{E}\left[\mathsf{E}[V - \mathsf{ask} | \pi^1, \text{I buy at } t=2] \middle| \pi^1 \right].$$

Similarly to Proposition 2, for any δ, the bid-ask spread in period 1 is larger than in period 2. We further find numerically that the L-shaped spread pattern becomes more pronounced as the probability of the public information release increases (δ decreases). When an information release is very unlikely (δ is large), volume is reverse L-shaped as before. However, when an information release is likely (δ is not too large), volume in period 1 exceeds volume in period 2. In other words, volume is L-shaped (or, arguably, reverse J-shaped) when traders perceive their private information to be very short-lived. Figure 5 illustrates the following finding.

Numerical Observation 2 (Explicit Discounting) *As δ decreases, $\mathsf{vol}^1 - \mathsf{vol}^2$ and $\mathsf{spread}^1 - \mathsf{spread}^2$ increase. Further, there exists a $\delta^\star$ such that $\mathsf{vol}^1 > \mathsf{vol}^2$ for all $\delta \leq \delta^\star$.*

[16]Our existence proof assumes $\delta = 1$, but can accommodate $\delta < 1$ with minor modifications.



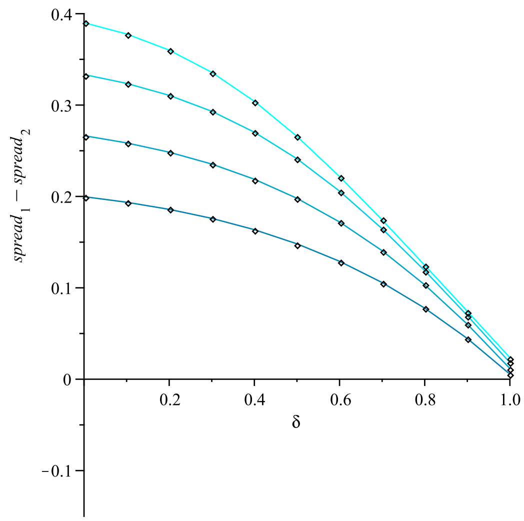


Figure 5: **Explicit Discounting: Comparative Static.** The probability of an informed trader is set to $\mu = .4$. Each line corresponds to the parameter of the quadratic quality distribution outlined in Appendix A; the parameters are $\theta \in \{-6, 0, 6, 12\}$. The competition parameter is set to $\alpha = 1$. The discount factor δ is on the horizontal axis. The *left panel* plots the difference of period 1 and 2 volumes, the *right panel* plots the difference in the period 1 and 2 bid-ask spreads.

Observation 1 from the last section illustrated that as competition increases, the difference in volumes, $\mathsf{vol}^1 - \mathsf{vol}^2$, increases. This insight extends for $\delta < 1$. If the probability of a public information release is low and volume in period 2 exceeds that in period 1, then competition causes the reverse L-shaped volume pattern to be less pronounced. However, if the probability of an information release is high and period 1 volume exceeds that in period 2, then increased competition enhances the L-shaped pattern.

6 Systematic Changes in the Information Structure

Our analysis of competition thus far has focussed on the expected number of traders. Traders, however, care not only for presence or absence of competition but also for the quality of information that the other trader may possess. We will now study how systematic changes in the distribution of information qualities affect observable variables.

These changes may occur when there is a persistent shift in the fraction of traders who are better informed or more capable at processing information. An example is an increase in analyst coverage for a stock, as this would improve the average trader's information. A stock may also attract a more informed clientele when it gets included in a major index, because major funds will add it to their portfolios. With such an inclusion, the company often faces additional disclosure requirements, further affecting the distribution of traders' signal qualities. Many of these changes are observable and the predicted impacts can be tested empirically. Additionally, the quality of analysts' earnings forecasts can serve as a proxy for the average information quality.

Formally, one would model improvements in information quality by shifts in the underlying distribution of qualities in the sense of *first order stochastic dominance* (FOSD). A systematic *improvement in information quality* corresponds to a situation in which the "new" quality distribution first-order stochastically dominates the "old" one so that under the new distribution, traders have systematically higher quality information. The analysis in this section is based on the quadratic signal quality distribution that is outlined in Appendix A. This class of distributions is parameterized by a parameter θ, which corresponds to the slope of the distribution's density. An increase in θ invokes a first order stochastic dominance shift. Figure 6 illustrates the following observation.

Numerical Observation 3 (Information Quality) *Fix $\delta = 1$. As the information quality improves systematically, we observe that $\mathsf{spread}^1, \mathsf{spread}^2$, $\mathsf{spread}^1 - \mathsf{spread}^2$, $\mathsf{vol}^1 + \mathsf{vol}^2$ and $\mathsf{vol}^1 - \mathsf{vol}^2$ increase.*

Observation 3 implies, in particular, that the L-shaped spread pattern becomes more pronounced, and that the reverse L-shaped volume pattern becomes less pronounced.

When information quality systematically increases, traders compete with on average better informed peers. In order to trade, they require higher quality information in both periods. At the same time, the concentration of traders with high quality information increases, yielding the intuition for the observation on volume.

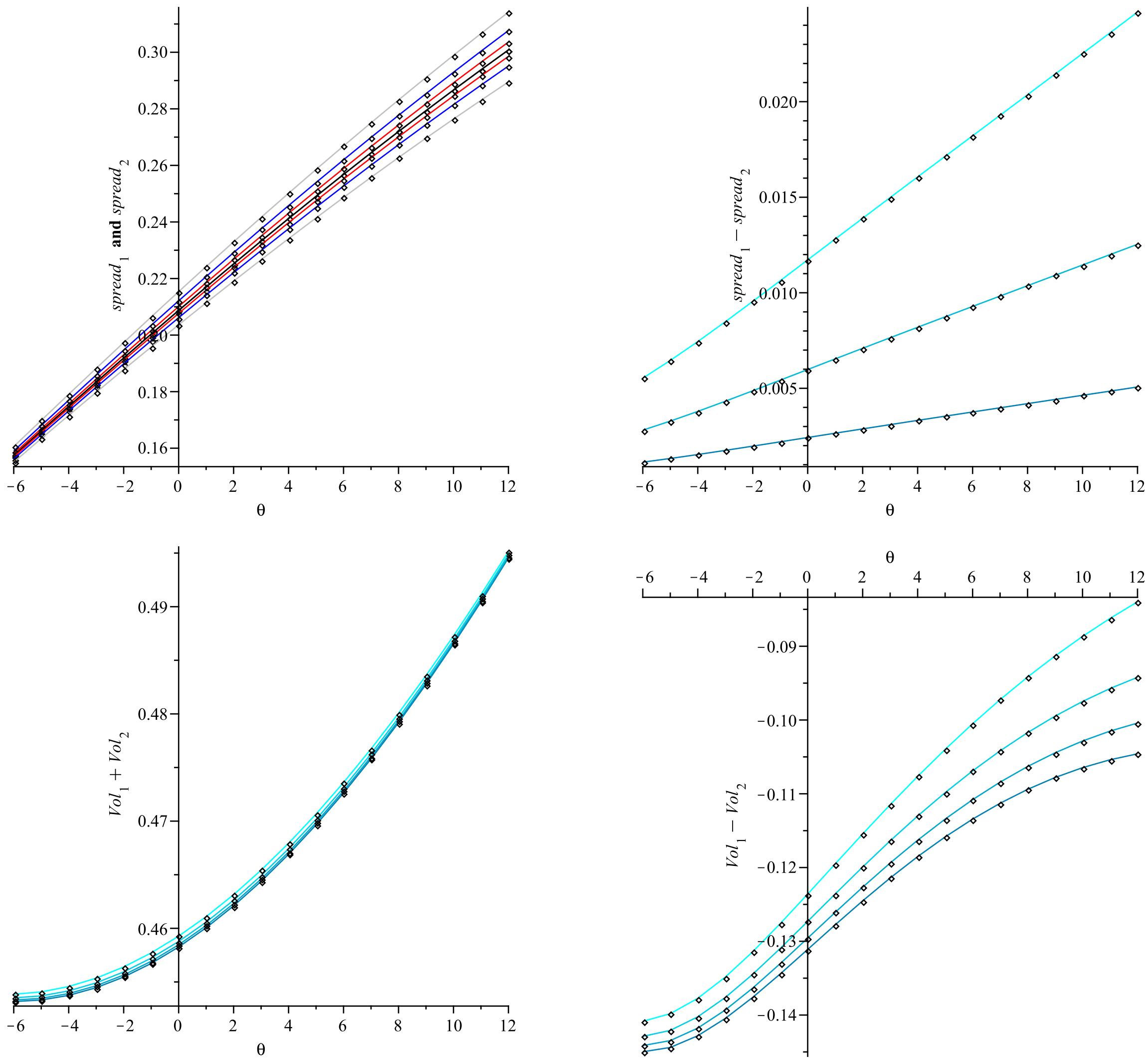


Figure 6: **Information Quality: Comparative Static.** The probability of an informed trader is set to $\mu = .4$. Each line in each panel is a function of the information quality parameter θ and corresponds to a level $\alpha \in \{0, .2, .5, 1\}$. The *top left panel* plots spread^1 (upper half of the lines) and spread^2 (lower half of the lines). The *top right panel* plots the difference of the period 1 and 2 spreads (it contains three lines as the difference in spreads for $\alpha = 0$ is zero). The *bottom left panel* plots total volume, the *bottom right panel* plots the difference in volumes for period 1 and 2. As can be seen, spreads, spread differences, total volumes, and volume differences all increase in θ.

7 Conclusion

The purpose of this study is to understand informed traders' timing decisions and the impact of these decisions on major economic variables. We predict that the heterogeneity in traders' informational advantages generates distinct intra-day volume and spread patterns. The predicted behavior is consistent with stylized facts that volume increases and spread declines toward the end of the trading day on most international stock exchanges.

The literature has identified other causes for variations in volume and transaction costs, such as discretionary liquidity traders (Admati and Pfleiderer (1988)) or the release of public information (Foster and Viswanathan (1990)). Our model is consistent with these causes, as including them would generally enhance the volume and spread patterns that we identify. We further provide novel predictions on how competition, possible releases of public information, and the informational environment affect the patterns in observables.

These predictions are facilitated by our choice of the underlying model. Analyzing a framework in the tradition of Glosten and Milgrom (1985) with a continuous signal structure allows us to study an uncertain number of traders, to explicitly describe bid-ask spreads, and to provide novel predictions on patterns in the probability of informed trading, a common measure of adverse selection costs (see, e.g., Hasbrouck (2007)).

A Appendix: Quality and Belief Distributions

The information structure used in this paper is as in Malinova and Park (2009). Financial market microstructure models with binary signals and states typically employ a constant common signal quality $q \in [^1\!/_2, 1]$, with $\mathsf{pr}(\text{signal} = h|V = 1) = \mathsf{pr}(\text{signal} = l|V = 0) = q$. This parameterization is easy to interpret, as a trader who receives a high signal h will update his prior in favor of the high liquidation value, $V = 1$, and a trader who receives a low signal l will update his prior in favor of $V = 0$. We thus use the conventional description of traders' information, with qualities $q \in [^1\!/_2, 1]$, in the main text.

As discussed in the main text, to facilitate the analysis, we map a vector of a trader's signal and its quality into a scalar continuous variable on $[0, 1]$, namely, the trader's private belief. To derive the distributions of traders' private beliefs, it is mathematically convenient to normalize the signal quality so that its domain coincides with that of the private belief. We will denote the distribution function of this normalized quality on $[0, 1]$ by G and its density by g, whereas the distribution and density functions of original qualities on $[{}^1\!/_2, 1]$ will be denoted by $\tilde{G}$ and $\tilde{g}$ respectively.

The normalization proceeds as follows. Without loss of generality, we employ the density function g that is symmetric around ${}^1\!/_2$. For $q \in [0, {}^1\!/_2]$, we then have $g(q) = \tilde{g}(1-q)/2$ and for $q \in [{}^1\!/_2, 1]$, we have $g(q) = \tilde{g}(q)/2$.

Under this specification, signal qualities q and $1-q$ are equally useful for the individual: if someone receives signal h and has quality ${}^1\!/_4$, then this signal has "the opposite meaning", i.e. it has the same meaning as receiving signal l with quality ${}^3\!/_4$. Signal qualities are assumed to be independent across agents and independent of the fundamental value V.

Beliefs are derived by Bayes Rule, given signals and signal qualities. Specifically, if a trader is told that his signal quality is q and receives a high signal h then his belief is $q/[q + (1-q)] = q$ (respectively $1-q$ if he receives a low signal l), because the prior is ${}^1\!/_2$. The belief π is thus held by people who receive signal h and quality $q = \pi$ and by those who receive signal l and quality $q = 1-\pi$. Consequently, the density of individuals with belief π is given by $f_1(\pi) = \pi[g(\pi)+g(1-\pi)]$ when $V = 1$ and analogously by $f_0(\pi) = (1-\pi)[g(\pi) + g(1-\pi)]$ when $V = 0$. Smith and Sorensen (2008) prove the following property of private beliefs (Lemma 2 in their paper):

Lemma 3 (Symmetric beliefs, Smith and Sorensen (2008)) *With the above signal quality structure, private belief distributions satisfy $F_1(\pi) = 1-F_0(1-\pi)$ for all $\pi \in (0, 1)$.*

Proof: Since $f_1(\pi) = \pi[g(\pi) + g(1-\pi)]$ and $f_0(\pi) = (1-\pi)[g(\pi) + g(1-\pi)]$, we have $f_1(\pi) = f_0(1-\pi)$. Then $F_1(\pi) = \int_0^\pi f_1(x)dx = \int_0^\pi f_0(1-x)dx = \int_{1-\pi}^1 f_0(x)dx = 1 - F_0(1-\pi)$. $\square$

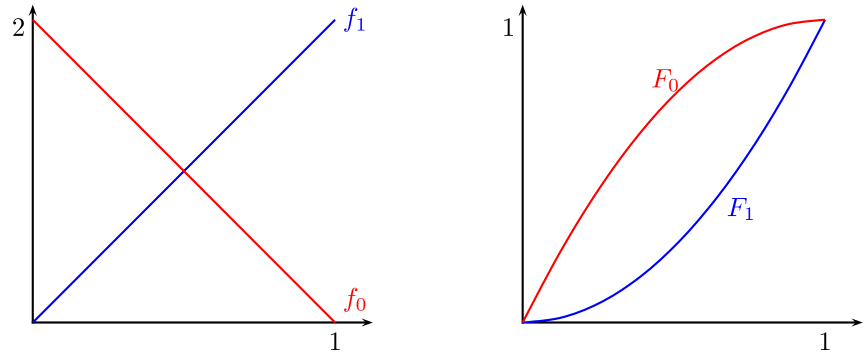


Figure 7: **Plots of belief densities and distributions.** *Left Panel:* The densities of beliefs for an example with uniformly distributed qualities. The densities for beliefs conditional on the true state being $V = 1$ and $V = 0$ respectively are $f_1(\pi) = 2\pi$ and $f_0(\pi) = 2(1-\pi)$; *Right Panel:* The corresponding conditional distribution functions: $F_1(\pi) = \pi^2$ and $F_0(\pi) = 2\pi - \pi^2$.

Belief densities obey the monotone likelihood ratio property as the following increases in π

$$\frac{f_1(\pi)}{f_0(\pi)} = \frac{\pi[g(\pi) + g(1-\pi)]}{(1-\pi)[g(\pi) + g(1-\pi)]} = \frac{\pi}{1-\pi}. \tag{5}$$

One can recover the distribution of qualities on $[{}^1\!/_2, 1]$, denoted by $\tilde{G}$, from G by combining qualities that yield the same beliefs for opposing signals (e.g $q = {}^1\!/_4$ and signal h is combined with $q = {}^3\!/_4$ and signal l). With symmetric g, $G({}^1\!/_2) = {}^1\!/_2$, and

$$\tilde{G}(q) = \int_{\frac{1}{2}}^{q} g(s)ds + \int_{1-q}^{\frac{1}{2}} g(s)ds = 2\int_{\frac{1}{2}}^{q} g(s)ds = 2G(q) - 2G({}^1\!/_2) = 2G(q) - 1.$$

An example of private beliefs. Figure 7 depicts an example where the signal quality q is uniformly distributed. The uniform distribution implies that the density of individuals with signals of quality $q \in [{}^1\!/_2, 1]$ is $\tilde{g}(q) = 2q$. When $V = 1$, private beliefs $\pi \geq {}^1\!/_2$ are held by traders who receive signal h of quality $q = \pi$, private beliefs $\pi \leq {}^1\!/_2$ are held by traders who receive signal l of quality $q = 1 - \pi$. Thus, when $V = 1$, the density of private beliefs π for $\pi \in [{}^1\!/_2, 1]$ is given by $f_1(\pi) = \mathsf{pr}(h|V = 1, q = \pi)\tilde{g}(q = \pi) = 2\pi$ and for $\pi \in [0, {}^1\!/_2]$ it is given by $f_1(\pi) = \mathsf{pr}(l|V = 1, q = 1-\pi)\tilde{g}(q = 1-\pi) = 2\pi$. Similarly, the density conditional on $V = 0$ is $f_0(\pi) = 2(1-\pi)$. The distributions of private beliefs are then $F_1(\pi) = \pi^2$ and $F_0(\pi) = 2\pi - \pi^2$. Figure 7 also illustrates that signals are informative:

recipients in favor of $V = 0$ are more likely to occur when $V = 0$ than when $V = 1$.

Signal quality distributions for simulations. In our numerical simulations we employed a quadratic quality distribution, the density of which is symmetric around 1/2.

$$g(q) = \theta \left(q - \frac{1}{2} \right)^2 - \frac{\theta}{12} + 1, \quad q \in [0, 1]. \tag{6}$$

The feasible parameter space for θ is $[-6, 12]$ and includes the uniform density for $\theta = 0$. Moreover, for $\theta' > \theta'$, $\tilde{G}_{\theta'}$ first order stochastically dominates $\tilde{G}_{\theta}$ as $\tilde{G}_{\theta'}(q) < \tilde{G}_{\theta}(q)$ for all $q \in [1/2, 1]$.

B Appendix: Omitted Proofs

B.1 Some General Results and Notation

We will first introduce some notation and establish basic results that facilitate the analysis and proofs of our main results.

B.1.1 General notation for all proofs

Using Bayes Rule and conditional independence of traders' private beliefs, an informed trader's expectation of the security's fundamental value, conditional his private information and the order flow, can be written as

$$\mathsf{E}[V|\pi, o^t] = \frac{\pi \mathsf{pr}(V = 1|o^t)}{\pi \mathsf{pr}(V = 1|o^t) + (1 - \pi)(1 - \mathsf{pr}(V = 1|o^t))}. \tag{7}$$

In other words, a trader's expectation of V can be expressed in terms of his private belief π and his prior $p = \mathsf{pr}(V = 1|o^t)$ (the probability that the value is $V = 1$, conditional on the order history but not on the trader's private belief), where the latter summarizes the information from the order flow that is relevant for estimating the fundamental value. In what follows, we will use $\mathsf{E}V[\pi; p]$ to denote a trader's expectation of the fundamental, conditional on this trader's private signal and his prior p.

Similarly, we will use function $a(\underline{\pi}, \overline{\pi}; p)$ to denote the liquidity provider's expectation of the security value, given prior p, conditional on a buy order that stems from either a noise trader drawn from a mass of size λ, or from an informed trader drawn from a mass of size μ and equipped with a private belief between $\underline{\pi}$ and $\overline{\pi}$. Conditional on the true value being $V = v$, the probability of such an order is $\beta_v(\underline{\pi}, \overline{\pi}) = \lambda + \mu(F_v(\overline{\pi}) - F_v(\underline{\pi}))$. Then, using Bayes Rule and rearranging,

$$a(\underline{\pi}, \overline{\pi}; p) = \left[1 + \frac{1-p}{p} \frac{\beta_0(\underline{\pi}, \overline{\pi})}{\beta_1(\underline{\pi}, \overline{\pi})}\right]^{-1} \tag{8}$$

This specification will allow us to compactly express the equilibrium ask prices. Further, we will use function $\pi^*(\overline{\pi}; p)$ to denote π that solves

$$\mathsf{E}V(\pi; p) = a(\pi, \overline{\pi}; p), \tag{9}$$

and we will use $\Pi(p)$ to denote π that solves

$$\mathsf{E}V(\pi^*(\pi; p); p) = a(\pi, 1; p). \tag{10}$$

Functions $\pi^*(\overline{\pi}; p)$ and $\Pi(p)$ will be useful in expressing the equilibrium thresholds and their bounds, and we study their properties in more detail in the next subsection.

B.1.2 Preliminary Properties

In what follows, it will often be mathematically convenient to express the private belief distributions F_1, F_0 in terms of the underlying quality distribution function G:

$$F_1(\pi) = 2\int_0^{\pi} s \cdot g(s)\ ds,\ F_0(\pi) = 2\int_0^{\pi} (1-s) \cdot g(s)\ ds \ \Rightarrow\ F_1(\pi) + F_0(\pi) = 2G(\pi), \tag{11}$$

and integrating by parts,

$$F_1(\pi) = 2\pi G(\pi) - 2\int_0^{\pi} G(s)\ ds. \tag{12}$$

Lemma 4 (Properties of the equilibrium thresholds)

(a) *For every $\overline{\pi}$ such that $1/2 < \overline{\pi} < 1$, there exists a unique $\pi \in (.5, \overline{\pi})$ that solves equation (9). This solution is independent of prior p: $\pi^*(\overline{\pi}; p) = \pi^*(\overline{\pi})$.*

(b) *$\pi^*(\overline{\pi})$ increases in $\overline{\pi}$: $\partial\pi^*/\partial\overline{\pi} > 0$.*

(c) *For fixed $\overline{\pi}$, $\pi = \pi^*(\overline{\pi})$ maximizes $a(\pi, \overline{\pi}; p)$. Further, $a(\pi, \overline{\pi}; p)$ increases in π for $\pi < \pi^*(\overline{\pi})$ and it decreases in π for $\pi > \pi^*(\overline{\pi})$.*

Proof of (a): Equation (9) can be rewritten as

$$\frac{\pi}{1-\pi} = \frac{\lambda + \mu(F_1(\overline{\pi}) - F_1(\pi))}{\lambda + \mu(F_0(\overline{\pi}) - F_0(\pi))}, \tag{13}$$

thus the solution does not depend on the prior p. Using (11) and (12), we rewrite (13) as

$$2\mu G(\overline{\pi})(\overline{\pi} - \pi) - 2\mu \int_{\pi}^{\overline{\pi}} G(s)\ ds - \lambda(2\pi - 1) = 0. \tag{14}$$

Denote the left hand side of the above equation by $\psi(\pi, \overline{\pi})$. Then

(i) $\psi(\pi, \overline{\pi})$ strictly decreases in π for $\pi \leq \overline{\pi}$: $\partial\psi/\partial\pi = -2\lambda - 2\mu(G(\overline{\pi}) - G(\pi)) < 0$;

(ii) at $\pi = 1/2$, $\psi(1/2, \overline{\pi}) = 2\mu G(\overline{\pi})(\overline{\pi} - 1/2) - 2\mu \int_{1/2}^{\overline{\pi}} G(s)\ ds > 0$;

(iii) at $\pi = \overline{\pi}$, $\psi(\overline{\pi}, \overline{\pi}) = -\lambda(2\overline{\pi} - 1) < 0$.

Steps $(i) - (iii)$ imply existence and uniqueness of $\pi^*(\overline{\pi})$.

Proof of (b): Applying the Implicit Function Theorem and differentiating both sides of equation (14) with respect to $\overline{\pi}$, and using g to denote the density function of qualities, we obtain

$$\frac{\partial \pi^*}{\partial \overline{\pi}} = \frac{2\mu g(\overline{\pi})(\overline{\pi} - \pi^*(\overline{\pi}))}{2\mu(G(\overline{\pi}) - G(\pi^*(\overline{\pi}))) + 2\lambda} > 0,$$

since $\pi^*(\overline{\pi}) \in (1/2, \overline{\pi})$ and G is increasing.

Proof of (c)**:** The first order condition for maximizing $a(\pi, \overline{\pi}; p)$ in π can be written as

$$\frac{\beta_1(\pi, \overline{\pi})}{\beta_0(\pi, \overline{\pi})} = \frac{\partial \beta_1(\pi, \overline{\pi})/\partial \pi}{\partial \beta_0(\pi, \overline{\pi})/\partial \pi} \quad \Leftrightarrow \quad \frac{\beta_1(\pi, \overline{\pi})}{\beta_0(\pi, \overline{\pi})} = \frac{\pi}{1 - \pi}, \tag{15}$$

where the last equality follows from equation (5). Observe that this last equality coincides with equation (13). Consequently, there exists a unique π that maximizes $a(\pi, \overline{\pi}; p)$ and this $\pi = \pi^*(\overline{\pi})$.

By (8), $a(\pi, \overline{\pi}; p)$ increases in π when $\beta_1(\pi, \overline{\pi})/\beta_0(\pi, \overline{\pi})$ increases in π. Using (5), (11), and (12), it can be shown that $(\partial/\partial\pi)(\beta_1(\pi, \overline{\pi})/\beta_0(\pi, \overline{\pi})) > 0$ when $\psi(\pi, \overline{\pi}) > 0$. The desired slopes then follow from part (a).

Lemma 5 *For every prior p, there exists $\Pi(p) \in (.5, 1)$ that solves equation (10). This solution is independent of p: $\Pi(p) = \Pi$.*

Proof: Equation (10) can be rewritten as

$$\frac{\pi^*(\pi; p)}{1 - \pi^*(\pi; p)} = \frac{\beta_1(\pi, 1)}{\beta_0(\pi, 1)}. \tag{16}$$

By Lemma 4, $\pi^*(\pi; p) = \pi^*(\pi)$, and thus the solution does not depend on the prior p. For the remainder of this proof, LHS refers to the left-hand side of (16) and RHS refers to the right-hand side of (16).

To prove that the solution exists and is unique, we apply Lemma 4 to observe that (i) for $\pi \leq \pi^*(1)$ LHS<RHS, as $\pi^*(\pi)/(1 - \pi^*(\pi)) < \pi/(1 - \pi) \leq \beta_1(\pi, 1)/\beta_0(\pi, 1)$; (ii) at $\pi = 1$ LHS>RHS, as as $\pi^*(\pi)/(1 - \pi^*(\pi)) > 1 = \beta_1(\pi, 1)/\beta_0(\pi, 1)$ and finally, (iii) LHS is increasing in π for all π, and RHS is decreasing in π for $\pi > \pi^*(1)$.

B.2 Existence: Proof of Theorem 1

The existence proof proceeds in four steps, by backward induction. We first show that for any given marginal buyer in period 1, π^1, there exists a unique marginal buyer in period 2, π^2, who is indifferent between trading and abstaining from trade (and thus prove Lemma 2). Step 2 verifies that monotone decision rules in period 2 are incentive-compatible. Step 3 shows existence of $\pi^1 \in (\pi^*(1), \Pi)$, and Step 4 verifies the incentive-compatibility of monotone decision rules in period 1.

Step 1: For all $\pi^1 \in [{}^1\!/_2, 1]$ there exists a unique period 2 marginal buyer, π^2, who is indifferent between submitting a buy order and abstaining from trade for *any* period 1 outcome (buy, sell or no trade). Further, $\pi^2 = \pi^*(\pi^1)$ and thus $\frac{d}{d\pi^1}\pi^2 > 0$.

Proof: Suppose first that there is a buy in period 1. The dealer then knows that this buy order came either from a noise trader or from an informed trader with a private belief between π^1 and 1. If a buy order also arrives in period 2, she will additionally learn that (i) there are 2 traders and (ii) the second trader is either a noise trader or an informed trader with a private belief between π^2 and π^1. Applying Bayes Rule, the ask price quoted in period 2, $\mathsf{ask}^2(\mathrm{B})$, can then be simplified to

$$\mathsf{ask}^2(\mathrm{B}) = \frac{\beta_1(\pi^1, 1)\beta_1(\pi^2, \pi^1)}{\beta_0(\pi^1, 1)\beta_0(\pi^2, \pi^1) + \beta_1(\pi^1, 1)\beta_1(\pi^2, \pi^1)} = a(\pi^2, \pi^1; p^1_{\mathsf{B}}),$$

where $p^1_{\mathsf{B}} = \beta_1(\pi^1, 1)/(\beta_1(\pi^1, 1) + \beta_0(\pi^1, 1))$. Likewise, conditionally on a buy order in period 1, trader π^2 updates his expectation to

$$\mathsf{E}[V|\mathrm{B} \text{ in } 1, \pi^2] = \frac{\pi^2\beta_1(\pi^1, 1)}{(1-\pi^2)\beta_0(\pi^1, 1) + \pi^2\beta_1(\pi^1, 1)} = \mathsf{E}V(\pi^2; p^1_{\mathsf{B}}).$$

The indifference condition for the marginal buyer is then $\mathsf{E}V(\pi^2; p^1_{\mathsf{B}}) = a(\pi^2, \pi^1; p^1_{\mathsf{B}})$, and the marginal buyer is given by $\pi^2 = \pi^*(\pi^1)$ by Lemma 4. The case with a sale in period 1 is analogous, and $\pi^2 = \pi^*(\pi^1)$.

Suppose now that there is no trade in period 1. A trader in period 2 must then account for the possibility that there may be a second block order, in which case he will be quoted an updated ask price and may elect to abstain from trading. Denote the initial quoted period 2 ask price by $\mathsf{ask}^2(\mathsf{NT})$ and the updated quotes, conditional on the other trader buy and sell orders, respectively, by $\mathsf{ask}^2_{\mathsf{B}}(\mathsf{NT})$ and $\mathsf{ask}^2_{\mathsf{S}}(\mathsf{NT})$. The zero expected profit condition for the marginal buyer π^2 can then be written as follows:

$$
\begin{aligned}
0 \;=\; & (1 - \mathsf{pr}(\text{B in 2}|\text{NT in 1}, \pi^2) - \mathsf{pr}(\text{S in 2}|\text{NT in 1}, \pi^2)) \times \\
& \qquad\qquad\qquad \left\{\mathsf{E}[V|\text{NT in 1, NT in 2}, \pi^2] - \mathsf{ask}^2(\mathsf{NT})\right\} \\
& + \mathsf{pr}(\text{B in 2}|\text{NT in 1}, \pi^2) \max\left\{0, \mathsf{E}[V|\text{NT in 1, B in 2}, \pi^2] - \mathsf{ask}^2_{\mathsf{B}}(\mathsf{NT})\right\} \\
& + \mathsf{pr}(\text{S in 2}|\text{NT in 1}, \pi^2) \max\left\{0, \mathsf{E}[V|\text{NT in 1, S in 2}, \pi^2] - \mathsf{ask}^2_{\mathsf{S}}(\mathsf{NT})\right\},
\end{aligned}
$$

where the expectations and probabilities are with respect to trader π^2's information set. When computing these, the trader conditions on his private belief being the marginal one as well as on the observed actions of the other trader (accounting also for the possibility of being alone in the market). The option to reconsider the trading decision in the event of a second block order has a non-negative value, as the trader may choose to abstain from trading upon hearing the updated quote. Thus, hypothetically, the marginal trader may be willing to accept losses at the initial quoted price and expect to profit in the event that there is a second block order. In what follows, we will show that this does not occur. In equilibrium, a trader will not change his decision, the value of the option to reconsider is zero, and the marginal buyer π^2 will be indifferent between trading and abstaining for any action of the second trader (as well as in the absence of the other trader). In the process, we will argue that this marginal buyer must satisfy $\pi^2 = \pi^*(\pi^1)$.

The quoted ask price, $\mathsf{ask}^2(\mathsf{NT})$, reflects the dealer's expectation of the fundamental conditional on (i) a buy order in period 2 and (ii) no other trade in either period. This occurs when (i) there is a single trader and he buys in period 2, or (ii) there are two traders, one of them buys in period 2 and the other elects to abstain from trading. In a symmetric equilibrium, the probability that a trader abstains from trading when $V = v$ is given by $F_v(\pi^2) - F_v(1 - \pi^2)$, and by Lemma 4 it is independent of the fundamental V. The quoted ask price in period 2 can then be simplified to $\mathsf{ask}^2(\mathsf{NT}) = a(\pi^2, \pi^1; 1/2)$.

Likewise, a trader in period 2 knows that his buy order will execute at the initial quoted ask price, $\mathsf{ask}^2(\mathsf{NT})$, when (i) he is alone in the market, or (ii) the second trader is present but chooses to abstain from trading. The probability of receiving the quoted ask price $\mathsf{ask}^2(\mathsf{NT})$ thus does not depend on the value of the fundamental, and the trader's conditional expectation is given by

$$\mathsf{E}[V|\text{NT in 1, NT in 2}, \pi^2] = \frac{\mathsf{pr}(\text{NT in 1, NT in 2}|V = 1)\pi^2}{\mathsf{pr}(\text{NT in 1, NT in 2}|\pi^2)} = \pi^2 = \mathsf{E}V(\pi^2; 1/2)$$

Lemma 4 then implies that any trader $\pi^2 > \pi^*(\pi^1)$ whose order is executed at the initial quoted price $\mathsf{ask}^2(\mathsf{NT})$ will make positive expected profits when he is assumed to be the period 2 marginal buyer; any trader $\pi^2 < \pi^*(\pi^1)$ will make negative trading profits in this scenario; and trader $\pi^2 = \pi^*(\pi^1)$ will make zero profits.

If two buy orders arrive in period 2, the dealer would quote an updated ask price. Denoting $p^2_{\mathsf{B}} = \beta_1(\pi^2, \pi^1)/(\beta_1(\pi^2, \pi^1) + \beta_0(\pi^2, \pi^1))$, this ask price can be written as $\mathsf{ask}^2_{\mathsf{B}}(\mathsf{NT}) = a(\pi^2, \pi^1; p^2_{\mathsf{B}})$. When receiving such quote, trader π^2 will learn that there is a second buyer in the market and will update his expectation of the fundamental to $\mathsf{E}[V|\text{NT in 1, B in 2}, \pi^2] = \mathsf{E}V(\pi^2; p^2_{\mathsf{B}})$

Lemma 4 then implies that any trader $\pi^2 > \pi^*(\pi^1)$ will accept the updated ask price $\mathsf{ask}^2_{\mathsf{B}}(\mathsf{NT})$ and make positive expected profits, any trader $\pi^2 < \pi^*(\pi^1)$ will reject the new price and make zero profits, and trader $\pi^2 = \pi^*(\pi^1)$ will be exactly indifferent between trading and abstaining. Analogously, the same decisions and profits obtain for the updated ask price in the event of a buy and a sale, $\mathsf{ask}^2_{\mathsf{S}}(\mathsf{NT})$.

We have thus shown that when trader π^2 submits a buy order and is assumed to be the marginal trader, then (i) when $\pi^2 > \pi^*(\pi^1)$, he will make positive expected profits for any realization of the other trader's actions (or in the absence of the other trader), (ii) when $\pi^2 < \pi^*(\pi^1)$, he will make negative trading profits if his order executes at the initial ask price and zero profits if he receives an updated quote (hence, negative profits in expectation), and (iii) when $\pi^2 = \pi^*(\pi^1)$, he will make zero profits in all cases. Consequently, $\pi^2 = \pi^*(\pi^1)$ is the unique marginal trader in period 2.

Step 2: Monotone decision rules in period 2 are incentive compatible in equilibrium: for given marginal traders $\pi^1, \pi^2 = \pi^*(\pi^1)$, ($i$) any trader $\pi > \pi^2$ who finds himself in period 2 will submit a buy order and will accept an updated price quote if presented with it; (ii) no trader $\pi < \pi^2$ will buy.

Proof: Observe first that the quoted price depends only on the *marginal* traders, thus all traders receive the same quotes. Next, by Step 1, for any realization of the trading history, these quotes coincide with the expectation of the marginal trader π^2, conditional on this history. Step 2 then follows as traders' expectations increase in private beliefs; see equation (1).

Step 3: There exists a period 1 marginal buyer, $\pi^1 \in (\pi^*(1), \Pi)$, who is indifferent between submitting a buy order in period 1 and delaying until period 2. Further, there does not exist a period 1 marginal buyer outside these bounds.

Proof: Analogously to the argument in Step 1, we can show that any trader $\pi^1 < \pi^*(1)$ would make negative expected profits from submitting a buy order in period 1, when he is assumed to be the marginal buyer in period 1. It is thus necessary that $\pi^1 \geq \pi^*(1)$. By the same argument, any trader $\pi^1 \geq \pi^*(1)$ will make non-negative profits in this scenario and will accept an updated price quote in period 1, if presented with it. Further, by Step 2, if trader π^1, who is assumed to be the marginal trader in period 1, delays trading until period 2, then he will make positive expected profits from submitting a buy order then and will always accept the updated ask quote, if presented with it, as $\pi^1 > \pi^*(\pi^1)$.

The marginal buyer π^1 in period 1 must be indifferent between submitting his buy order in period 1 and submitting it in period 2. The indifference condition for this trader is $\mathsf{E}[V - \mathsf{ask}^1|\text{I submit } \mathsf{B} \text{ at } t = 1,\, \pi^1] = \mathsf{E}[\mathsf{E}[V - \mathsf{ask}^2|\text{I submit } \mathsf{B} \text{ at } t = 2,\, \pi^1, H_1]|\pi^1]$. Since the marginal trader always accepts the quote once the order is submitted, we can use the Law of Iterated Expectations and rewrite the indifference condition as

$$\mathsf{E}[\mathsf{ask}^1|\text{I buy in 1, } \pi^1] - \mathsf{E}[\mathsf{E}[\mathsf{ask}^2|\text{I buy in 2, } \pi^1, H_1]|\pi^1] = 0. \tag{17}$$

When computing these conditional expectations trader π^1 accounts, in particular, (i) for himself being the marginal buyer in period 1 and (ii) for trader $\pi^2 = \pi^*(\pi^1)$ being the marginal buyer in period 2.

Denote the left-hand side of (17) by $\xi(\pi^1)$. We will show (a) that $\xi(\pi^*(1)) > 0$ and (b) that $\xi(\Pi) < 0$. The desired existence of π^1 then follows by continuity. We will then show (c) that there does not exist a marginal buyer $\pi^1 > \Pi$.

Part (a) *We show that* $\xi(\pi^*(1)) > 0$. Set $\pi^1 = \pi^*(1)$. We can show, similarly to the proof of Step 1, that $\mathsf{E}V(\pi^*(1), 1/2) = \mathsf{E}[\mathsf{ask}^1|\text{I buy in 1, } \pi^*(1)]$. Thus showing that $\xi(\pi^*(1)) > 0$ is equivalent to showing that

$$\mathsf{E}V(\pi^*(1), 1/2) - \mathsf{E}[\mathsf{E}[\mathsf{ask}^2|\text{I buy in 2, } \pi^*(1), H_1]|\pi^*(1)] > 0. \tag{18}$$

We will denote the realized *total* number of buys and sales in period 2 by b_2, s_2, respectively, and use H_1 for the period 1 transaction history. Using the Law of Iterated Expectations and the fact that $\pi^*(1)$ always trades at an updated quote, we write (18) as

$$\sum_{H_1}\sum_{b_2,s_2} \mathsf{pr}(b_2, s_2, H_1)(\mathsf{E}[V|b_2, s_2, H_1; \text{I buy in } 2, \pi^*(1), H_1] - \mathsf{E}^{\mathsf{M}}[V|b_2, s_2, H_1]) > 0,$$

The above inequality is satisfied, because, by Step 1, the market maker's conditional expectation of the fundamental $\mathsf{E}^{\mathsf{M}}[V|b_2, s_2, H_1]$ coincides with that of the period 2 marginal trader π^2, and the latter is below the expectation of the marginal trader in period 1, $\mathsf{E}[V|b_2, s_2, H_1; \text{I buy in } 2, \pi^*(1), H_1]$, by Lemma 4, since $\pi^2(\pi^1) < \pi^1$, for *any* history.

Part (b) We show that $\xi(\Pi) < 0$. Set the marginal buyer in period 1 to be $\pi^1 = \Pi$, and the marginal buyer in period 2 to be $\pi^2 = \pi^*(\Pi)$. The price impact of a trader's action is the same in both periods:

$$\frac{\beta_1(\Pi, 1)}{\beta_0(\Pi, 1)} = \frac{\beta_1(\pi^*(\Pi), \Pi)}{\beta_0(\pi^*(\Pi), \Pi)} = \frac{\pi^*(\Pi)}{1 - \pi^*(\Pi)}$$

In a symmetric equilibrium, this implies, in particular, that the quoted (initial and updated) ask prices depend only on the total number of buy and sale block orders but not on the time of the order submission. Consequently, the only scenario, in which the price that the trader pays will depend on the period that he submits his order in, is when (i) there are 2 traders and (ii) the second trader trades (buys or sells) in period 2.

To shorten the exposition, we will omit the arguments from the function β_v and use β_v^t to denote the probability of a buy in period t, conditional on $V = v$ (although, the price impacts coincide, $\beta_1^1/\beta_0^1 = \beta_1^2/\beta_0^2$, the conditional probabilities do not, $\beta_v^1 \neq \beta_v^2$). To show that $\xi(\Pi) < 0$, we need to show that the marginal buyer $\pi^1 = \Pi$ expects to pay more in period 2 than in period 1, conditional on there being 2 traders and the second trader trading in period 2 (where we use $\beta_1^1/\beta_0^1 = \beta_1^2/\beta_0^2$):

$$\frac{\Pi\beta_1^2 + (1-\Pi)\beta_0^2}{\beta_1^2 + \beta_0^2}\left[1 + \left(\frac{\beta_0^2}{\beta_1^2}\right)^2\right]^{-1} + \frac{(1-\Pi)\beta_1^2 + \Pi\beta_0^2}{\beta_1^2 + \beta_0^2}\frac{1}{2} > \left[1 + \frac{\beta_0^2}{\beta_0^2}\right]^{-1}, \tag{19}$$

Observe that (i) the price that the trader pays conditional on the other trader buying in period 2, $[1+(\beta_0^2/\beta_1^2)^2]^{-1}$, exceeds that paid conditional on the other trader selling, $1/2$; (ii) probabilities of the other trader buying and selling in period 2, conditional on him trading in period 2 sum to 1; (*iii*) the marginal trader's private belief Π exceeds $\beta_1^2/(\beta_0^2+\beta_1^2)$; and (*iv*) $\beta_1^2/(\beta_0^2+\beta_1^2) > 1/2$. Replacing Π with $\beta_1^2/(\beta_0^2+\beta_1^2)$ will thus decrease the weight on the larger ask price and increase the weight on the smaller one (keeping the sum of these weights at 1), thereby decreasing the left-hand side of (19):

$$\text{LHS of (19)} > \frac{(\beta_1^2)^2+(\beta_0^2)^2}{(\beta_1^2+\beta_0^2)^2}\left[1+\left(\frac{\beta_0^2}{\beta_1^2}\right)^2\right]^{-1}+\frac{2\beta_0^2\beta_1^2}{(\beta_1^2+\beta_0^2)^2}\frac{1}{2}=\left[1+\frac{\beta_0^2}{\beta_0^2}\right]^{-1}.$$

Part (c): We show that the marginal trader in period 1 must be $\pi^1 < \Pi$. We will argue that $\xi(\pi^1) < 0$ for $\pi^1 > \Pi$. When $\pi^1 > \Pi$, the price impact of a trader's buy order is stronger in period 2 than in period 1. Consequently, conditional on being alone in the market or on the other trader abstaining from trading, trader π^1 expects to pay a higher price in period 2 than in period 1. We can also show, similarly to the argument at the beginning of Step 1, that conditional on the second trader being present and trading in period 1, trader π^1 will prefer to submit his buy order in period 1. Hence, to argue that $\xi(\pi^1) < 0$, it suffices to show that trader π^1 expects to pay a higher price in period 2, conditional on the other trader being present and trading in period 2, or that the left-hand side of (19) exceeds $[1+(\beta_0^1/\beta_1^1)^2]^{-1}$. This follows from Part (*b*), as $[1+(\beta_0^2/\beta_1^2)^2]^{-1} > [1+(\beta_0^1/\beta_1^1)^2]^{-1}$.

Step 4: The monotone decision rules in period 1 are incentive compatible in equilibrium: given marginal traders π^1 and $\pi^2 = \pi^*(\pi^1)$, (*i*) any trader $\pi > \pi^1$ buys in period 1 and accepts an updated quote, if presented with it; (*ii*) no trader $\pi < \pi^1$ will buy in period 1.

Proof: First, by the proof of Step 3, any trader $\pi > \pi^1$ will accept the updated quote in either period. To show that the monotone decision rules are incentive compatible, it thus suffices to show that for given marginal traders $\{\pi^1, \pi^2\}$, the difference between the price that trader π expects to pay if he submits his buy order in period 1 and that if he submits his buy order in period 2 is decreasing in π. (A trader will buy in period 1 only if this difference is negative).

The quotes that trader π receives are determined by the marginal types and thus do not depend on π. Denote the difference between the (updated) ask quote in period 1 and the (initial) ask quote in period 2, conditional on the other trader buying in period 1 by $\Delta(\mathsf{B}_1)$; denote this difference, conditional on the other trader selling in period 1 by $\Delta(\mathsf{S}_1)$; likewise for $\Delta(\mathsf{B}_2)$ and $\Delta(\mathsf{S}_2)$; and denote the difference between the initial price quotes in periods 1 and 2, conditional on no other trader acting, by $\Delta(\mathsf{NT})$. Continue to use β_v^t for the probability of a buy in period t, conditional on $V = v$, given marginal buyers $\pi^1, \pi^2 = \pi^*(\pi^1)$, and use γ to denote the (equilibrium) probability that a trader abstains from trading. The expected price difference for trader π is then given by

$$\begin{aligned}\mathsf{E}[\mathsf{ask}^1 | \text{I trade in 1, } \pi] - \mathsf{E}[\mathsf{ask}^2 | \text{I trade in 2, } \pi] = (1-\alpha+\alpha\gamma)\Delta(\mathsf{NT}) \\ +\alpha(\pi\beta_1^1 + (1-\pi)\beta_0^1)\Delta(\mathsf{B}_1) + \alpha((1-\pi)\beta_1^1 + \pi\beta_0^1)\Delta(\mathsf{S}_1) \\ +\alpha(\pi\beta_1^2 + (1-\pi)\beta_0^2)\Delta(\mathsf{B}_2) + \alpha((1-\pi)\beta_1^2 + \pi\beta_0^2)\Delta(\mathsf{S}_2),\end{aligned}$$

where trader π conditions on traders π^1 and $\pi^*(\pi^1)$ being the marginal buyers. Differentiating the right-hand side, this difference is decreasing in π when

$$(\beta_1^1 - \beta_0^1)(\Delta(\mathsf{B}_1) - \Delta(\mathsf{S}_1)) + (\beta_1^2 - \beta_0^2)(\Delta(\mathsf{B}_2) - \Delta(\mathsf{S}_2)) < 0. \tag{20}$$

Observe that (i) $\beta_1^t > \beta_0^t$ for $t = 1, 2$; (ii) $\Delta(\mathsf{B}_2) - \Delta(\mathsf{S}_2) = 1/2 - [1 + (\beta_0^2/\beta_1^2)^2]^{-1} < 0$, as the quote in period 1 is unaffected by the other trader's action in period 2. It thus suffices to show that $\Delta(\mathsf{B}_1) - \Delta(\mathsf{S}_1) < 0$. In what follows, we will compress the notation further and denote the period t likelihood of a buy for $V = 0$ relative to $V = 1$ by $l^t = \beta_0^t/\beta_1^t$. Using this, $\Delta(\mathsf{B}_1) = 1/(1 + (l^1)^2) - 1/(1 + l^1 l^2)$, and $\Delta(\mathsf{S}_1) = 1/2 - 1/(1 + l^2/l^1)$. Step 3 implies that, in equilibrium $1 > l^2 > l^1$, and it thus suffices to prove that

$$\frac{1}{1+(l^1)^2} - \frac{1}{2} < \frac{1}{1+l^1 l^2} - \frac{1}{1+l^2/l^1} \text{ for } l^2 > l^1. \tag{21}$$

Observe that (i) the left-hand side of (21) is independent of l^2, and (ii) at $l^2 = l^1$, the left-hand side coincides with the right-hand-side. To prove inequality (21), it thus suffices to prove that its right-hand side increases in l^2 for fixed l^1. Differentiating the right-hand side with respect to l^2 and rearranging, the derivative is positive if and only if $1+l^1l^2 > l^1+l^2$. The latter inequality is true by Chebyshev's inequality.[17]

B.3 Competition: Proof of Proposition 1

For each level of competition α, we will use $\xi(\pi^1, \alpha)$ to denote the left-hand side of equation (17) and $\pi^1(\alpha)$ to denote the equilibrium period 1 marginal buyer.[18] By Lemma 2, the second period marginal buyer $\pi^2(\pi^1)$ increases in π^1 and the marginal seller is symmetric, $1-\pi^2(\pi^1)$. Consequently, (i) the volume maximizing equilibrium obtains for the lowest π^1 that solves $\xi(\pi^1, \alpha) = 0$, and (ii) it suffices to prove that π^1 decreases in α. Take $\tilde{\alpha} > \alpha$. To show that π^1 decreases in α, it suffices to show that $\xi(\pi^1(\alpha), \tilde{\alpha}) < 0$, or that $\frac{\partial \xi}{\partial \alpha}|_{\pi^1=\pi^1(\alpha)} < 0$. For, since $\xi(\pi^*(1); \tilde{\alpha}) > 0$ by Step 3 of Theorem 1, by continuity, there must exist $\tilde{\pi}^1 \in (\pi^*(1), \pi^1(\alpha))$ such that $\xi(\tilde{\pi}^1, \tilde{\alpha}) = 0$.

We will now show that $\frac{\partial \xi}{\partial \alpha}|_{\pi^1=\pi^1(\alpha)} < 0$. First, rewrite equation (17) as

$$\begin{aligned} 0 \;=\; & \alpha(\mathsf{E}[\mathsf{ask}^1|\text{I buy in 1, } \pi^1, 2 \text{ traders}] - \mathsf{E}[\mathsf{E}[\mathsf{ask}^2|\text{I buy in 2, } \pi^1, H_1]|\pi^1, 2 \text{ traders}]) \qquad (22) \\ & +(1-\alpha)(\mathsf{E}[\mathsf{ask}^1|\text{I buy in 1, } \pi^1, 1 \text{ trader}] - \mathsf{E}[\mathsf{E}[\mathsf{ask}^2|\text{I buy in 2, } \pi^1, H_1]|\pi^1, 1 \text{ trader}]). \end{aligned}$$

Observe that when the trader is alone in the market, he can perfectly forecast both ask prices, and the second term on the right-hand side is $(1-\alpha)(a(\pi^2, \pi^1; {}^1\!/\!_2) - a(\pi^1, 1; {}^1\!/\!_2))$. This term is positive at $\pi^1 = \pi^1(\alpha)$ because, by Step 3 (c) of the proof of Theorem 1, $\pi^1(\alpha) < \Pi$ and thus (i) by Lemma 4, $a(\pi^1, 1; {}^1\!/\!_2)) > a(\Pi, 1; {}^1\!/\!_2))$, and (ii) by Lemmas 1 and 4, $a(\pi^2, \pi^1; {}^1\!/\!_2) = \mathsf{E}V(\pi^*(\pi^1); {}^1\!/\!_2) < \mathsf{E}V(\pi^*(\Pi); {}^1\!/\!_2) = a(\Pi, 1; {}^1\!/\!_2)$. For (22) to hold, the first

[17] Chebyshev's inequality states that if $a_1 \geq a_2 \geq \ldots \geq a_n$ and $b_1 \geq b_2 \geq \ldots \geq b_n$, then $\frac{1}{n}\sum_{k=1}^n a_k b_k \geq \left(\frac{1}{n}\sum_{k=1}^n a_k\right)\left(\frac{1}{n}\sum_{k=1}^n b_k\right)$. Here we use $n = 2$, $a_1 = 1, a_2 = l^1$ and $b_1 = 1, b_2 = l^2$.

[18] In the proof of Theorem 1, we omitted dependence on α.

term in (22) must then be negative at $\pi^1 = \pi^1(\alpha)$. Finally, observe that, for a fixed π^1, conditional expectations in equation (22) are independent of α.

The partial derivative $\frac{\partial\xi}{\partial\alpha}|_{\pi^1=\pi^1(\alpha)}$ is the partial derivative of the right hand side of (22). The above discussion implies that it equals the first term in parentheses of (22) minus the second term in parentheses of (22), and that $\frac{\partial\xi}{\partial\alpha}|_{\pi^1=\pi^1(\alpha)} < 0$.

B.4 Spreads: Proof of Proposition 2

Recall that the spread is defined as the difference between the ask and the bid price, and to facilitate the comparison, in period 2 we look at the spread that obtains conditional on no transactions in period 1. In equilibrium, $\mathsf{bid}^t = 1 - \mathsf{ask}^t$ and $\mathsf{spread}^t = 2\mathsf{ask}^t - 1$, and it suffices to show that (i) $\mathsf{ask}^1 > \mathsf{ask}^2$ and (ii) ask^1 increases in α and ask^2 decreases in α.

To see (i) observe that $\mathsf{ask}^1 = a(\pi^1, 1; 1/2)$ and, conditional on no transaction in period 1, $\mathsf{ask}^2 = a(\pi^2, \pi^1; 1/2)$. The inequality then follows by the same argument as in the proof of Proposition 1.

To see (ii) observe first that both π^1 and π^2 decrease in α by Proposition 1. By Step 3 of the proof of Theorem 1, $\pi^1 > \pi^*(1)$; Lemma 4 then implies that $a(\pi^1, 1; 1/2)$ decreases in π^1 and thus increases in α. Further, by Step 1 of the same proof, $a(\pi^2(\pi^1), \pi^1; 1/2) = \pi^*(\pi^1)$; it increases in π^1 by Lemma 4 and thus decreases in α.

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Result	Economic description	Parameter	Observable Reaction
Proposition 1 (Market Participation)	competition increases	$\alpha \nearrow$	$\mathsf{vol}^1 \nearrow$ $\mathsf{vol}^1 + \mathsf{vol}^2 \nearrow$
Proposition 2 (Spreads)	irrespective of the level of competition	$\forall \alpha > 0$	$\mathsf{spread}^1 > \mathsf{spread}^2$
	competition increases	$\alpha \nearrow$	$\mathsf{spread}^1 \nearrow$ $\mathsf{spread}^2 \searrow$ $\mathsf{spread}^1 - \mathsf{spread}^2 \nearrow$
Numerical Observation 1 (Volume)	irrespective of the level of competition	$\forall \alpha \geq 0$	$\mathsf{vol}^2 > \mathsf{vol}^1$
	competition increases	$\alpha \nearrow$	$\mathsf{vol}^1 - \mathsf{vol}^2 \nearrow$
Proposition 3 (PIN)	PIN^t follows vol^t	$\mathsf{vol}^1 > \mathsf{vol}^2$ $\mathsf{vol}^1 < \mathsf{vol}^2$	$\mathsf{PIN}^1 > \mathsf{PIN}^2$ $\mathsf{PIN}^1 < \mathsf{PIN}^2$
Numerical Observation 2 (Public Information Release)	release of public information $\nearrow$	$\delta \searrow$	$\mathsf{vol}^1 - \mathsf{vol}^2 \nearrow$ $\mathsf{spread}^1 - \mathsf{spread}^2 \nearrow$
	L-shaped volume	$\exists \delta^\star$ s.t. $\forall \delta \leq \delta^\star$	$\mathsf{vol}^1 > \mathsf{vol}^2$
Numerical Observation 3 (Information Quality)	systematic information quality improvement	$\theta \nearrow$	$\mathsf{spread}^1 \nearrow$ $\mathsf{spread}^2 \nearrow$ $\mathsf{spread}^1 - \mathsf{spread}^2 \nearrow$ $\mathsf{vol}^1 + \mathsf{vol}^2 \nearrow$ $\mathsf{vol}^1 - \mathsf{vol}^2 \nearrow$

Table 1: **Summary of the Model's Empirical Predictions**