On-the-Job Training, Limited Commitment, and Firing Costs†

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Abstract

We develop a model of optimal dynamic labor contracts in which firms provide training to workers, but workers cannot credibly commit to stay with the firm. The training is in the form of skills which can profitably be used in other job matches. In the model, wage rates rise with seniority and separation rates are higher for low skilled workers than for high skilled workers. These observations are consistent with the evidence. We show that firing costs (*i*) increase the average duration of employer-worker matches, (*ii*) reduce the outside value of the worker, and (*iii*) increase the level of training provided by the firm. This mechanism has the opposite effect on measured productivity than the selection mechanism of search models with endogenous separation, where firing costs reduce the average quality of active matches. Our model provides a rationale for the positive relationship observed between the level of employment protection and TFP across European countries.

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1 Introduction

The process of human capital accumulation constitutes a fundamental source of wage growth for people over the life cycle, as well as a source of productivity increases for the aggregate economy over time.

In this paper we focus on firm-provided training in general human capital. There is substantial evidence that firms promote, provide, and pay for training investment in general skills for the workers (see the evidence in the OECD Employment Outlook, 2003). We also focus on the impact of training on productivity and wages. The evidence shows that there are substantial returns to labor market experience and tenure on the job. This return to experience for instance implies that wages more than double in the first 20 years of labor market experience.

We borrow from a large theoretical literature that emphasizes the role of credit market imperfections and limited commitment on the under provision of firm-sponsored training. (See for instance Becker, 1975; Acemoglu and Pischke, 1999a, and the OECD Employment Outlook: 2003, chapter 5.) This literature emphasizes the possibility that frictions in the labor market may provide incentives for firm-provided training in general skills. Moreover, credit market imperfections for investment in general human capital and limited commitment from the part of the worker (hold-up problem) are identified as key factors leading to under investment in training. The empirical evidence, while conclusive about the extent and importance of firm-provided training in most developed countries, is not conclusive about the connection between labor market policies/institutions and investment in training. Given the importance of training for productivity and wage growth, we develop a quantitative model of on-the-job human capital accumulation to study the impact of labor market policy, in particular employment protection (firing costs) and unemployment benefits.

We develop a theory of optimal dynamic labor contracts in which firms provide training to workers, but workers cannot credibly commit to stay with the firm. The training is in the form of skills which can profitably be used in other job matches (general human capital). We build on the labor-matching framework of Mortensen and Pissarides (1994) where workers spend time unemployed searching for jobs. Our model also borrows from the human capital theory of Becker (1975) in that workers cannot borrow to finance investment in training, and from models of long-term labor contracts as in Harris and Holmstrom (1988) with risk averse workers, although we abstract from asymmetric information problems. A final theoretical source includes models with idiosyncratic productivity shocks and endogenous separation, as Hopenhayn and Rogerson (1993) and Lagos (2005).

To make a quantitative assessment of the effects of labor market policies on training we calibrate our benchmark economy with no firing costs to U.S. data on unemployment, vacancies, training expenditures over GDP, wage inequality, and replacement unemployment benefits. We compare the implications of our benchmark economy with limited commitment to the situation where there is perfect commitment from the part of the workers. In our framework, limited commitment from the part of the worker implies that job matches are destroyed at a higher rate than in the case of perfect commitment. This higher job destruction rate implies that the average duration of a job-worker relationship is lower, the unemployment rate is higher, and investment in training is lower. Therefore, limited commitment implies under-provision of training and a lower aggregate human capital stock relative to the perfect commitment case. These effects are consistent with the emphasis in the existing literature (see for instance the OECD Jobs Study, 1994, the OECD Employment Outlook, 2003 and the references therein).

An important question is whether labor market policy can restore incentives for investment in training. We assess the quantitative implications of labor market policy in our framework by studying the effects of firing costs and unemployment benefits. We analyze the impact on the steady-state equilibrium of the model of changing firing costs from zero to one average yearly wage, and changing replacement unemployment benefits from 24% percent of mean wage in the benchmark economy to 54% percent.

Our results suggest than firing costs have a positive impact on the average duration of matches, reducing endogenous job destruction and the unemployment rate (although the last effect is ambiguous, and quantitatively small). This is the result of a selection mechanism, as in Lagos (2005), where firms with lower productivity stay in the market when firing is costly. But, in addition, firing costs do increase the incentives for training, increasing the average level of human capital in the economy. We find that unemployment benefits in our model unambiguously reduce the duration of employment, increases the unemployment rate, and reduce training.

Some of our results on training differ sharply with the existing literature mainly because labor market policy has non-trivial effects on the job separation rate in the economy and because, with risk averse workers, firms find it costly in terms of insurance to satisfy incentives using steep wage profiles over states and over time. These two features of our model are abstracted from in the existing literature.

In Acemoglu and Pischke (1999a) firing costs compress the wage distribution, making the experience or tenure profile of wages less steep. This compression in wages allows the firm to recover part of the cost of training. In our model, training investment increases with firing costs even though the wage distribution is more unequal. The reason for this result in our model is that whereas the job separation rate in the benchmark economy is similar for high and low human capital workers, in the firing cost economy the job separation rate is much higher for low human capital workers, tilting the returns to human capital investment in favor of more training. This channel is absent in Acemoglu and Pischke (1999a).

Our result is also related to the work of Jansen (1998) where firing costs increase the amount of training. However, Jansen assumes exogenous differences in the job separation rates of high and low human capital workers, while in our model these separation rates are endogenous and responsive to policy. Moreover, in Jansen's theory, as emphasized in Acemoglu and Pischke (1999a), firing taxes compress the wage distribution. Because job separation rates are exogenous in Jansen (1998) and Acemoglu and Pischke (1999a), the implications of firing costs on unemployment are also different than ours.

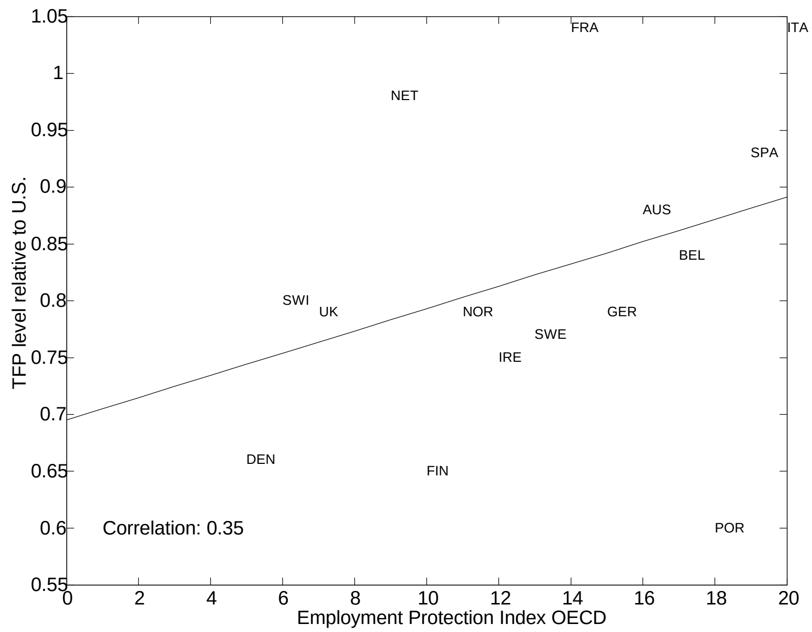
Our paper is also related to a recent literature that connects labor market policies with measured total factor productivity (TFP) differences across countries (see Lagos, 2005) In this literature, the selection effect implies that firing costs make less efficient matches to stay longer in the market, reducing average TFP. This mechanism abstracts from on-the-job training. While our model preserves the negative impact of firing costs on the quality of matches in the market, investment in training has the opposite effect, making the comparison of measured TFP across countries less straightforward. The empirical evidence is inconclusive regarding the relationship between labor market policies and measured TFP. As Figure 1 shows, at least within a set of European countries, an index of employment protection policies is positively correlated with measured TFP differences.

The structure of the paper is as follows. In the next section, we describe the economic environment and characterize some properties of the optimal labor contract. In section 3 we present the calibration of the model, and in section 4 we describe the main results of the numerical experiment. Section 5 concludes.

2 Economic Environment

We develop a model of optimal dynamic labor contracts with frictions in which firms provide training in general skills to workers, but workers cannot credibly commit to stay with the firm. We consider variations to the institutional environment where firms face a resource cost of turnover (firing cost) and workers have access to home production when unemployed

Figure 1: Employment Protection and Measured TFP



(unemployment benefits). In what follows we describe the economic environment in detail.

2.1 General Description

We assume that time is discrete but there are an infinite number of periods. The economy is populated by a large number (mass one) of ex-ante homogeneous workers that face exponential life, i.e., every period with probability η a worker dies and is replaced by a new-born worker. There is also a large number (mass one) of risk-neutral firms.

Preferences and Endowments Workers are risk averse. They have preferences over stochastic consumption sequences described by the following utility function

$$E_0 \sum_{t=0}^{\infty} [\beta(1-\eta)]^t u[c(t)].$$

where u is a strictly concave function.

Workers are endowed with $h(0)$ units of (general) human capital at the beginning of life (date 0). We assume that the human capital level of a worker can take only a finite number n of values: $h(t) \in \{h_1, h_2, ..., h_n\}$. A new-born workers start their life with the lowest level of human capital, i.e., $h(0) = h_1$.

Technologies At each date there is a single good produced. Production takes place in production units that consist of a worker and a firm. At each date, firms post vacancies and unemployed workers search for jobs. Firms can post any number of vacancies at a cost ϕ per vacancy in terms of the output good and a vacancy lasts for one period. Frictions in the labor market are summarized by an exogenous function $m(s, u)$ describing the number of matches formed in each period, where s is the number of vacancies posted and u the mass of unemployed workers searching for jobs. We denote by p the probability that a worker finds a job, and q the probability that a vacancy is filled.

A production unit with idiosyncratic productivity $A \in \left[0, \bar{A}\right]$ and a worker with general human capital h_j produces $AF(h_j)$ units of output. A firm can invest in the (general) human capital of the worker. Human capital is general in the sense that the worker can profitably use it in other firms. If a non-negative amount of the output good ψx is invested in training of a worker with human capital h_j, the return of training is uncertain: With probability $\rho(x)$ the human capital of the worker increases in the following period to h_{j+1}, otherwise it remains constant.

We assume that a new production unit always starts with the highest productivity $A = \bar{A}$. We also assume that A remains constant for a firm as long as the worker's human capital is constant. However, in the period in which training is successful and the worker upgrades her human capital, a new A is drawn from a Markov process: with probability λ, the firm's productivity remains constant and with probability $(1-\lambda)$ it is drawn from an i.i.d cumulative distribution function $G(A)$ over $[0, \bar{A}]$. One interpretation for this productivity shock is as an indicator of the quality of the match, which changes randomly whenever the human capital of the worker changes.

We assume that firms can borrow or lend from an outside party at the exogenous interest rate $\bar{\imath} \equiv \frac{1}{\beta(1-\eta)} - 1$. Workers cannot borrow or save.

Labor Contract If matched at date t, a worker with (general) human capital $h(t)$ and a firm write a contract to start producing at date $t+1$. The contract specifies sequences of contingent plans (contingent on the histories of human capital and productivity) for:

1. Wage rates (or consumption of the worker)

$$\left\{c\left(s \left| \left\{h\left(z\right), A\left(z\right)\right\}_{z=t+1}^{s}\right.\right)\right\}_{s=t+1}^{\infty}$$

2. Investment in training

$$\left\{x\left(s\left|\{h(z), A(z)\}_{z=t+1}^{s}\right.\right)\right\}_{s=t+1}^{\infty}$$

Notice that since the outcome of training is stochastic the contract does not specify directly a sequence for human capital $\{h(s)\}_{s=t+1}^{\infty}$.

Workers cannot credibly commit to stay in the firm. At the end of each period they can quit, in which case they become unemployed for the next period and start searching for a new job. There is not on-the-job search.

Firms can commit to enforce the contract. However, at the beginning of each period they face an exogenous probability γ of going out of business, in which case the firm disappears and the worker becomes unemployed and starts searching for a new job. If no exogenous separation occurs in the period, then firms are still potentially subject to productivity shocks. We assume that endogenous separation occurs if the realized productivity is such that the match is no longer profitable, in the sense that the present value of profits cannot cover both the firm and the worker's reservation values.

The optimal contract is designed to maximize the value of a matched vacancy for the firm (expected discounted present value of profits Π) subject to a participation constraint for the worker and an initial (expected) value promised to the worker v^{new}. This initial value is determined as the solution to the following Nash bargaining problem:

$$\max_{v}\left\{\Pi\left(\bar{A}, v\right)^{\theta}\left(v-v^{un}\right)^{1-\theta}\right\},$$

where $\theta \in (0,1)$ represents the firm's bargaining power and v^{un} is the value of an unemployed worker.

Institutional Features As a first attempt to model unemployment benefits, we assume that unemployed workers produce $\bar{b}$ units of the output good from home production. The human capital of unemployed workers is transferable among jobs (general training), and does not depreciate over time. We also model firing costs $\bar{f}$ as a loss of resources incurred by firms in the event of endogenous separation. These resource costs are death weight losses for the economy.

2.2 Recursive Optimal Contract

We focus on a stationary equilibrium of our environment and use recursive methods to characterize and solve for the optimal labor contract. At the beginning of the period, the individual state variables for the firm are described by:

- j (standing for h_j): human capital of the worker.
- A: productivity of the firm.
- v: expected and discounted value promised to the worker.

First, the firm observes the realization of the exogenous separation shock. If no exogenous separation occurs in this period, then given the state the firm chooses whether to stay in the market or to exit. Let's denote by $e \in \{0, 1\}$ this choice, with $e = 1$ meaning that the firm chooses to exit and the match is destroyed. In this case, the firm pays the firing cost $\overline{f}$, and the worker becomes unemployed. Exit occurs if:

$$\Pi_j^{e=0}\left(A, v_j^{un}\right) < -\overline{f}$$

this is, if the present value of the match is no longer profitable enough to provide both parties at least their reservation value.

If the firm stays ($e = 0$), then it decides the wage rate (or consumption) for the worker c, the amount of investment in training x, and the next period value promised to the worker v' (contingent on the state tomorrow), in order to solve the Bellman equation:

$$\Pi_j^{e=0}(A, v) = \max_{c, x, v'_j, v'_{j+1}(A')} \left\{ AF(h_j) - \psi x - c + \beta(1-\eta)(1-\gamma)\widehat{\Pi}_j\left(A, x, v'_j, v'_{j+1}(A')\right) \right\}$$

$$\begin{aligned} s.t \qquad & u(c) + \beta(1-\eta)\widehat{U}_j\left(A, x, v'_j, v'_{j+1}(A')\right) = v \\ & v'_j \geq v_j^{un}, \qquad v'_{j+1}(A') \geq v_{j+1}^{un}, \qquad \forall A' \in \left[0, \overline{A}\right], \\ & c, x \geq 0. \end{aligned} \tag{1}$$

The first constraint is the promise keeping constraint for the firm, while the next line includes all the participation constraints for the worker (for each possible state tomorrow). The continuation value for the firm is defined as

$$\widehat{\Pi}_j\left(A, x, v_j', v_{j+1}'(\bullet)\right) \equiv (1-\rho(x))\,\Pi_j^{e=0}(A, v_j')$$

$$+\rho(x)\left[\lambda\Pi_{j+1}(A, v_{j+1}'(A)) + (1-\lambda)\int_0^{\overline{A}} \Pi_{j+1}(A', v_{j+1}'(A'))dG(A')\right],$$

with $\rho(x) = 0$ if $j = n$ to take care for the cases in which worker's human capital has reached its maximum levels, and

$$\Pi_j(A, v) = \begin{cases} \Pi_j^{e=0}(A, v) & , \text{ if } \Pi_j^{e=0}\left(A, v_j^{un}\right) \geq -\overline{f} \\ -\overline{f} & , \text{ if } \Pi_j^{e=0}\left(A, v_j^{un}\right) < -\overline{f} \end{cases}$$

Similarly, we define the continuation value for the worker

$$\widehat{U}_j\left(A, x, v_j', v_{j+1}'(A')\right) \equiv (1-\rho(x))\left[(1-\gamma)\,v_j' + \gamma v_j^{un}\right]$$

$$+\rho(x)\left[(1-\gamma)\left(\lambda v_{j+1}'(A) + (1-\lambda)\int_0^{\overline{A}} v_{j+1}'(A')\,dG(A')\right) + \gamma v_{j+1}^{un}\right]$$

with

$$v_{j+1}'(A) = v_{j+1}^{un}, \qquad \text{if } \Pi_{j+1}^{e=0}\left(A, v_{j+1}^{un}\right) < -\overline{f}$$

Note that, in this recursive setup, an optimal contract corresponds to the equilibrium decision rules $g_j^c(A, v)$, $g_j^x(A, v)$, $g_j^e(A, v)$, $g_j^{v_j'}(A, v)$ and $g_j^{v_{j+1}'(A')}(A, v)$ for each state, while the initial value for a new worker corresponds to the equilibrium value v_j^{new}, for each $j \in \{1, ..., n\}$. With these elements, an optimal path for consumption, training, human capital and exit can be obtained applying recursively the optimal decision rules, starting from an initial value (corresponding to a given initial level of human capital) and resolving the uncertainty at each period.

2.3 Stationary Recursive Equilibrium

A stationary recursive equilibrium for this economy is a list of functions $\Pi_j^{e=0}(A, v)$, $g_j^c(A, v)$, $g_j^x(A, v)$, $g_j^e(A, v)$, $g_j^{v_j'}(A, v)$, $g_j^{v_{j+1}'(A')}(A, v)$, for each $j \in \{1, ..., n\}$, size n vectors v^{un}, v^{new}, probabilities p, q, numbers u, s, and invariant distributions μ_j^{un}, $\mu_j^{em}(A, v)$, for each $j \in \{1, ..., n\}$, such that:

1. The value of a standing firm matched with a worker with human capital h_j, $\forall j \in \{1, ..., n\}$, and productivity A, $\forall A \in \left[0, \overline{A}\right]$, solves the Bellman equation (1). In addition, $g_j^c(A,v)$, $g_j^x(A,v)$, $g_j^e(A,v)$, $g_j^{v'_j}(A,v)$, $g_j^{v'_{j+1}(A')}(A,v)$ are the optimal policy rules for this problem.

2. The value of a unemployed worker with human capital h_j is given by:

$$v_j^{un} = u\left(\bar{b}\right) + \beta\left(1-\eta\right)\left[pv_j^{new} + (1-p)v_j^{un}\right].$$

3. The value of a new worker with human capital h_j solves the Nash bargaining problem:

$$v_j^{new} = \arg\max_v \left\{\Pi_j\left(\overline{A}, v\right)^{\theta}\left(v - v_j^{un}\right)^{1-\theta}\right\},$$

$$s.t \qquad \Pi_j\left(\overline{A}, v\right) \geq 0, \qquad v \geq v_j^{un}.$$

4. A zero profit condition for posting a vacancy holds, i.e.,

$$\beta(1-\eta)q\sum_{j=1}^{n}\Pi_j\left(\overline{A}, v_j^{new}\right)\mu_j^{un} = \phi.$$

5. The probabilities of finding a job and filling a vacancy are obtained using the matching function:

$$p = \frac{m(u,s)}{u} \qquad q = \frac{m(u,s)}{s}.$$

6. The invariant distribution of unemployed workers at the beginning of the period satisfies:

$$\begin{aligned}\mu_j^{un} &= (1-\eta)(1-p)\mu_j^{un} \\ &+(1-\eta)\int_0^{\overline{A}}\int_V (1-\rho(g_j^x(A,v)))\left(\gamma + (1-\gamma)\,g_j^e(A,v)\right)\mu_j^{em}(A,v)dvdA \\ &+(1-\eta)\int_0^{\overline{A}}\int_V \rho(g_{j-1}^x(A,v))\Big[\gamma + (1-\gamma)\,\lambda g_j^e(A, g_{j-1}^{v'_j(A)}(A,v)) \\ &+ (1-\gamma)(1-\lambda)\int_0^{\overline{A}} g_j^e(A', g_{j-1}^{v'_j(A')}(A,v))dG(A')\Big]\mu_{j-1}^{em}(A,v)dvdA\end{aligned}$$

for each $j \in \{2, ..., n\}$. The corresponding law of motion for unemployed workers with human capital h_1 is:

$$\mu_1^{un} = \eta + (1-p)(1-\eta)\mu_1^{un}$$

$$+(1-\eta)\int_0^{\overline{A}}\int_V (1-\rho(g_1^x(A,v)))\,(\gamma + (1-\gamma)\,g_1^e(A,v))\,\mu_1^{em}(A,v)dvdA$$

7. The invariant distribution of employed workers at the beginning of period satisfies:

$$\mu_j^{em}(A,v) = (1-\eta)\,pI_{\left\{A=\overline{A},v=v_j^{new}\right\}}\mu_j^{un} + (1-\eta)\,(1-\gamma)\left(1-g_j^e(A,v)\right)$$

$$\times\left[\int_{\left\{\widetilde{v}\in V\middle|g_{j-1}^{v'_{j-1}}(A,\widetilde{v})=v\right\}}\left(1-\rho\left(g_j^x\,(A,\widetilde{v})\right)\right)\mu_j^{em}(A,\widetilde{v})d\widetilde{v}\right.$$

$$+\lambda\int_{\left\{\widetilde{v}\in V\middle|g_{j-1}^{v'_j(A)}(A,\widetilde{v})=v\right\}}\rho\left(g_{j-1}^x\,(A,\widetilde{v})\right)\mu_{j-1}^{em}(A,\widetilde{v})d\widetilde{v}$$

$$\left.(1-\lambda)\int_0^{\overline{A}}\int_{\left\{\widetilde{v}\in V\middle|g_{j-1}^{v'_j(A)}\left(\widetilde{A},\widetilde{v}\right)=v\right\}}\rho\left(g_{j-1}^x\left(\widetilde{A},\widetilde{v}\right)\right)\mu_{j-1}^{em}(\widetilde{A},\widetilde{v})d\widetilde{v}dG\left(\widetilde{A}\right)\right]$$

for each $j\in\{2,...,n\}$, where I is the indicator function. The corresponding law of motion for employed workers with human capital h_1 is:

$$\mu_1^{em}(A,v) = (1-\eta)\,pI_{\left\{A=\overline{A},v=v_1^{new}\right\}}\mu_1^{un}$$

$$+(1-\eta)\,(1-\gamma)\,(1-g_1^e(A,v))\int_{\left\{\widetilde{v}\in V\middle|g_1^{v'_1}(A,\widetilde{v})=v\right\}}(1-\rho(g_1^x(A,\widetilde{v})))\mu_1^{em}(A,\widetilde{v})d\widetilde{v}$$

2.4 Properties of the Optimal Contract

Given the equilibrium values v_j^{un}, v_j^{new}, and the probability p of finding a match, we can characterize the properties of the optimal contract using firm´s first order conditions. We summarize now some of these properties, which are formally analyzed in the Technical Appendix.

Threshold Productivity We can easily show that the value of continuing a match is strictly increasing in the match current productivity. Since the value of destroying the match does not depend on productivity, the exit problem has a simple solution: A match will be destroyed ($e = 1$) if and only if the current productivity draw lies below a threshold level $A_j^* \in [0, \overline{A}]$ satisfying

$$\Pi_j^{e=0}\left(A_j^*, v_j^{un}\right) = -\overline{f}$$

Of course, $A_j^* = 0$ if $\Pi_j^{e=0}\left(0, v_j^{un}\right) > -\overline{f}$, and $A_j^* = \overline{A}$ if $\Pi_j^{e=0}\left(\overline{A}, v_j^{un}\right) < -\overline{f}$.

Insurance and Constant Promised Value Since the worker is risk averse and the firm has access to an unlimited borrowing/lending technology, the cheapest way for a firm to deliver a given promised value is by a constant wage (consumption) profile. This implies that, independently on the productivity of the match:

$$g_j^{v_j'}(A, v) = v$$

$\forall A \in [0, \overline{A}]$ and $\forall v \geq v_j^{un}$, as it can be shown from firm's first order conditions. This is, if the worker does not increase her human capital from one period to the other (because the firm did not spend in training or because training was not successful), and if the match is not destroyed, the promised value to the worker remains constant. This is how the long term contract between the firm and the worker provides for insurance.

Incentives and Outside Option With full commitment, the firm always provides for a flat wage profile, as stated in the previous point. However, this solution might not be feasible without commitment from the worker's side, since the worker faces an increasing outside option given by the accumulation of general human capital. Trading off incentives vs. insurance, the firm provides the worker with a constant wage whenever this option satisfies the participation constraint for the worker. But if the worker's outside option increases to a level higher than the current promised value (because of successful training), the firm promises the worker exactly such outside value, i.e., the participation constraint binds. As long as the initial promised value is not too high, then the contract will deliver the worker a steep wage profile which follows the worker's outside option.

Distribution of Wages As a corollary form the previous two properties, we obtain a wage distribution with the following porperties. First, all workers with human capital h_{j+1} earn at least as much as each worker with the previous level of human capital h_j. In other words, wages are non-decreasing in worker´s human capital, independently of the productivity of the match and the promised value. Second, considering only workers with the same level of human capital h_j, the distribution of wages puts a positive mass in only two points: (i) workers with a promised value equal to their ouside option v_j^{un}; and (ii) workers with a promised value higher than their outside option. Again, wages for each group do not depend on the match productivity nor on the promised value, and workers from the second group have a higher wage than workers from the first group.

3 A Quantitative Economy

To obtain quantitative predictions for the model, we first restrict preferences and technologies to specific functional forms. We choose a log utility function $u(c) = \log(c)$ and a linear production function of the worker-job match $AF(h) = Ah$. The probability of increasing human capital through training is assumed to follow the strictly concave function:

$$\rho(x) = \frac{x}{1+x},$$

with $\rho(x) = 0$ and $\lim_{x\to\infty} \rho(x) = 1$. Finally, we choose a Cobb-Douglas matching function,

$$m(u,s) = Bu^{\varkappa}s^{1-\varkappa},$$

exhibiting constant returns to scale.

With respect to the levels of human capital h, we use an equally spaced grid of n_h points $\{h_1, h_2, ..., h_{n_h}\}$. We normalize the initial human capital to one $h_1 = 1$, and denote by $\overline{h}$ the maximum level. For simplicity, in this version we restrict the set of possible human capital levels to two. We also assume a uniform distribution for the match specific productivity shock between 0 and $\bar{A} = 15$, with persistence $\lambda = 0.75$, and perform sensitivity analysis on these two parameters.

3.1 Preliminary Calibration

We still need to provide values for the following 13 parameters: β, η (preferences), $\overline{h}$, ψ (training return and cost), λ, γ (productivity), B, $\varkappa$ (matching function), θ (Nash bargaining), ϕ (cost of vacancy), $\overline{b}$ (unemployment benefits) and $\bar{f}$ (firing cost). Our calibration strategy is to restrict parameter values of our benchmark economy with no firing taxes ($\bar{f} = 0$) to the U.S. data in the 1990's. We restrict our model period to be one quarter. We have then 12 parameters to choose, as summarized in Table 1.

A set of parameter values are chosen without solving the model. We assume $\beta = 0.99$ implying an annual rate of time preference of 4%, and $\eta = 0.006$ consistent with an expected

Table 1: List of Parameters

Parameters Selected with no Empirical Counterpart	
θ	Firm's bargaining power
$\bar{A}$	Highest level of match productivity
λ	Persistence in match productivity
Parameters Calibrated without Solving the Model	
β	Discount factor
η	Probability of dying
B	Scale in matching function
$\varkappa$	Unemployment elasticity in matching function
Parameters Selected by Solving the Model	
γ	Rate of exogenous match destruction
$\bar{h}$	Highest level of human capital
ψ	Cost of training
ϕ	Cost of posting a vacancy
$\bar{b}$	Unemployment benefits

average working life of 40 years.

We follow the approach in Erosa, et al. (2001) to calibrate the matching technology. First, we use the estimate of the elasticity of unemployed workers in the matching function in Blanchard and Diamond (1989). Using monthly data for the U.S. these authors estimate $\varkappa = 0.4$. Note that this number is close to the value used by Mortensen and Pissarides (1994) in their analysis of unemployment in a labor-matching framework. Second, van Ours and Ridders (1992) document an average duration of a vacancy in the U.S. economy of 45 days. Using this target and our specification of the matching function, we compute the probability that a vacancy is matched within a quarter as:

$$q = 1 - \left(1 - \frac{1}{45}\right)^{90} = 0.8677.$$

Third, the average duration of unemployment reported in the 2005 U.S. Economic Report of the President for the 1990 to 2000 period is 12 weeks. Using this target, we calculate the probability of being matched for an unemployed worker in a quarter to be:

$$p = 1 - \left(1 - \frac{1}{84}\right)^{90} = 0.6597.$$

Hence, since $q = B\left(\frac{u}{s}\right)^{\gamma}$ and $p = B\left(\frac{u}{s}\right)^{1-\gamma}$, we use the above values of p and q to get

$u/s = 1.3153$ and $B = 0.7776$. In the procedure described below, we choose the cost of a vacancy to match the unemployment to vacancy ratio. Given B above, this will satisfy our targets for p and q.

The last parameter that we set without solving the model is the bargaining power of firms in wage setting. There is very little empirical discipline in choosing this parameter. Therefore, we proceed by setting $\theta = 0.5$ which is the value used in Mortensen and Pissarides (1994) and subsequent literature in applied search models, but we perform sensitivity analysis for a range of values for θ.

The remaining five parameters: $\overline{h}$, γ, ϕ, ψ, and $\bar{b}$ are chosen simultaneously so that the stationary equilibrium for the benchmark economy matches the following observations for the U.S. economy:

1. An unemployment/vacancy ratio of 1.3153 as discussed above.
2. An unemployment rate of 5.4% as the average for the 1990-2000 period from the OECD Employment Outlook (2005).[1]
3. A replacement unemployment benefits rate of 24% as documented in the OECD Jobs Study (1994) for the U.S.
4. An average training expenditure to output ratio of 4%, as reported in Mincer (1993).
5. Earnings inequality for male workers (ratio of percentiles 90/10) of 5.3 from Jones and Weinberg (2000), obtained using CPS data.

The calibration exercise implies solving numerically the model for different sets of parameters ($\overline{h}$, γ, ϕ, ψ, and $\bar{b}$), and choose the set for which the statistics obtained from the equilibrium are closer to the targets. The numerical method to compute the decision rules and invariant distributions is described in detail in the Technical Appendix.

The parameters obtained from the calibration exercise are presented in Table 2. We match very close the calibration targets. If anything, the model generates slightly less wage inequality than in the data.

[1] Since in a stationary equilibrium the number of jobs created equals the number of jobs destroyed, this unemployment rate together with the value of p derived before imply a quarterly rate of job destruction of

$$p\left(\frac{u}{1-u}\right) = 0.6597\left(\frac{0.054}{1-0.054}\right) = 0.037$$

above the quarterly job destruction rate of 2.5% reported in Davis, Haltiwanger, and Schuh (1996) for the U.S. economy.

Table 2: Parameters Calibrated for the Benchmark Economy

Target	Data	Model	Parameter	Value
Unemployment rate (%)	5.40	5.47	γ	0.031
Unemployment/vacancy ratio	1.32	1.33	ϕ	157
Replacement rate for benefits (%)	24.0	24.1	$\bar{b}$	22.8
Training expenditures over output (%)	4.00	3.94	ψ	159
Range of wages (90/10 decile)	5.30	5.01	$\overline{h}$	7.65

4 Numerical Results

First, we present in details the results for the benchmark economy. To gain further intuition on the behavior of the model, we plot the decision rules and compare them to the case of perfect commitment. We also compute some relevant statistics from the invariant distributions and simulate the economy for a large number of firms and workers to obtain time profiles for the main variables. Second, we perform the experiments of changing the firing cost and unemployment benefits to various levels of protection, and show how the results obtained for the benchmark economy change. In particular, we are interested in the aggregate level of human capital employed as a measure of the impact of training.

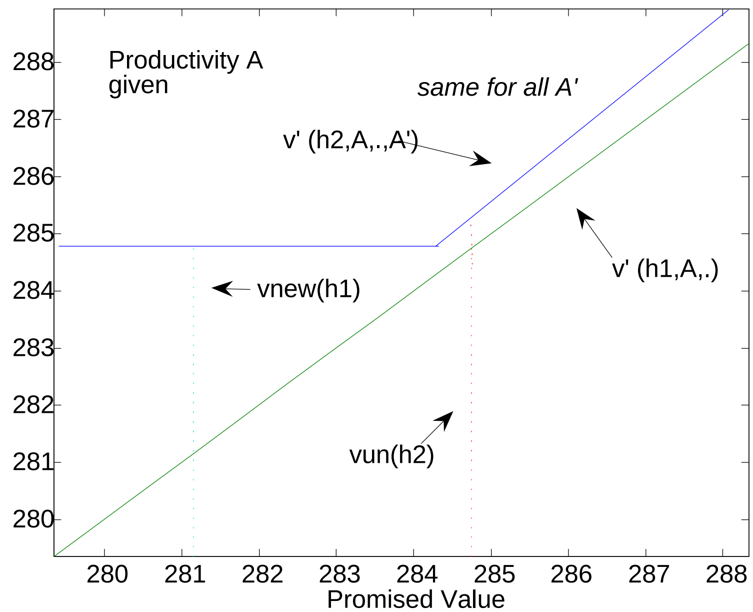
4.1 The Benchmark Economy

4.1.1 Policy Rules

To understand the role of limited commitment in the model it is key to analyze the policy rule for next period promised value. Figure 2 shows in detail this policy as a function of the current promised value for a worker with low human capital h_1 and given productivity $A > A_1^*$. The range of promised values start at v_1^{un} since no old or new worker would have accepted less than that. As explained before, the policy function $g_1^{v_1'}(A, v)$ coincides with the $45°$ line. However, if the worker improves his human capital, for low values of v the participation constraint binds while for values above the threshold v^* the firm promises a value for next period above the current value. Graphically, the policy function for $g_1^{v_2'(A')}(A, v)$ is an horizontal line up to v^*, followed by an increasing function lying above the $45°$ line. As discussed before, this function is the same for all $A' > A_1^*$.

Notice than in this example the cutoff value is strictly between the unemployment values for workers with low and high human capital. More importantly, the value of a new worker with low human capital v_1^{new} is below the cut-off value v^*. This means that the par-

Figure 2: Policy Rule for Next Period Promised Value



ticipation constraint binds for firms hiring low skilled workers, distorting their consumption and training decisions compared to a situation with no commitment problems.

The last point becomes clear by observing the policy rules for consumption and training as a function of the current promised value and for a given productivity, presented in Figure 3. The solid line represents a worker with the low level of human capital h_1, while the dotted line is for a worker with the highest human capital h_2. As expected, the decision rules for consumption and training are increasing in worker's promised value, with a kink at the cut-off value v^* below which the participation constraint for workers who increase their human capital binds. Note that consumption for both types of human capital is similar when the participation constraint does not bind; otherwise, the commitment problem kicks in reducing current consumption and increasing future consumption (steep wage profile). When the participation constraint binds, the provision of training is sub-optimal as well. We also verify that the value of the firm (present value of profits) for both types of workers is decreasing in the promised value and concave.

We finally show in Figure 4 the same policy rules as functions of the current productivity, keeping the current promised value constant. We verify that consumption and next period promised values do not depend on current productivity, due to the insurance motive.

Figure 3: Policy Rules as a function of Promised Value

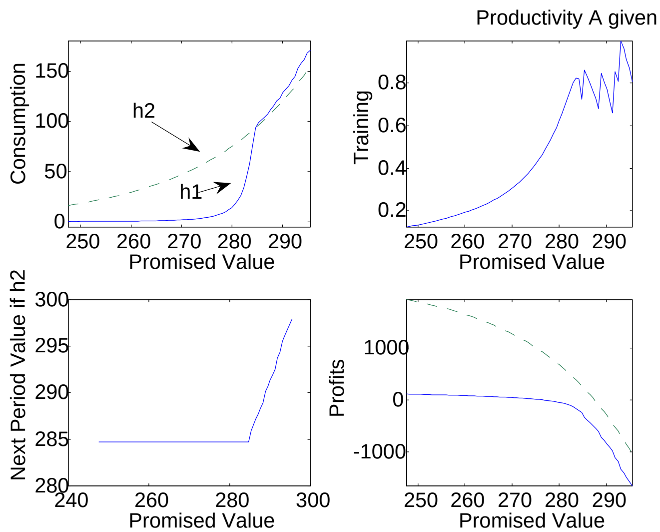


Figure 4: Policy Rules as a function of Productivity

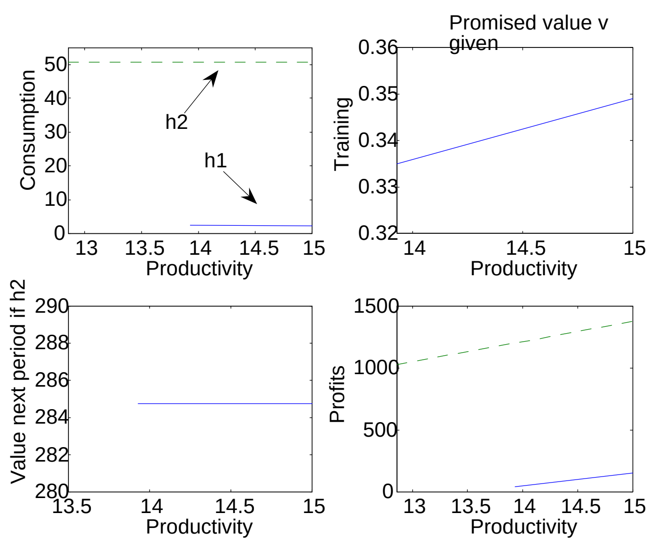


Table 3: Statistics from the Invariant Distribution

	Benchmark	Perfect Commitment
Value unemployed worker		
- low human capital	279.3	280.1
- high human capital	284.7	286.7
Value new worker		
- low human capital	281.1	281.9
- high human capital	287.1	288.5
Threshold Productivity		
- low human capital	13.93	12.86
- high human capital	12.23	11.63
Unemployment rate (percent)	5.48	4.69
Aggregate Human Capital		
- employed	6.35	6.47
- unemployed	0.31	0.37
Range of wage distr.		
- D9/D1 ratio	4.96	1.42

However, the training decision depends positively on current productivity which, given the persistence of the productivity shock, affects the returns to training. We also verify that the value of the firm for both types of workers is increasing in productivity.

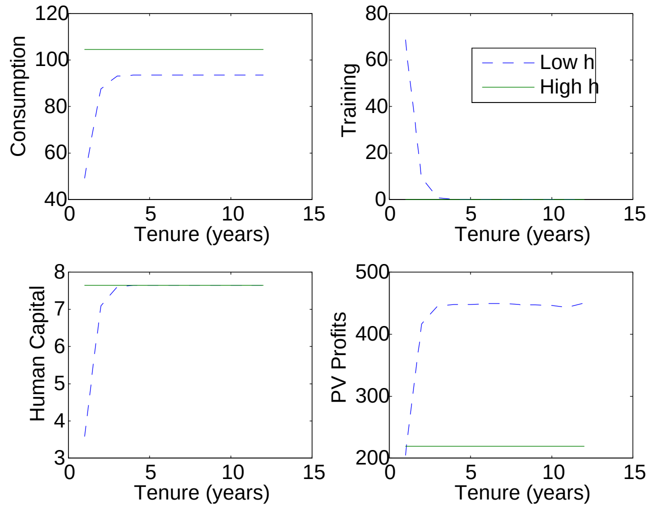
4.1.2 Equilibrium Aggregate Values

Table 3 show some statistics obtained from the invariant distributions of the benchmark economy and compares them with the case of perfect commitment (obtained solving the same model but ignoring the participation constraints). The first four lines correspond to the value of an unemployed and a new worker with low and high human capital. As expected, these values are higher under perfect commitment since profits are also bigger (the firm is optimizing over a larger set of options).

The third and fourth lines show the cut-off productivity level A_j^* for the two levels of human capital. Both are lower with limited commitment (again, because profits are lower), meaning than the probability of endogenous firing is higher. The commitment problem then induces separation and reduces the average tenure of workers. This is why the unemployment rate is lower under perfect commitment, as seen in the fifth line.

The next two lines present the aggregate level of human capital, employed and unem-

Figure 5: Tenure Profiles for New Workers with Different Levels of Human Capital



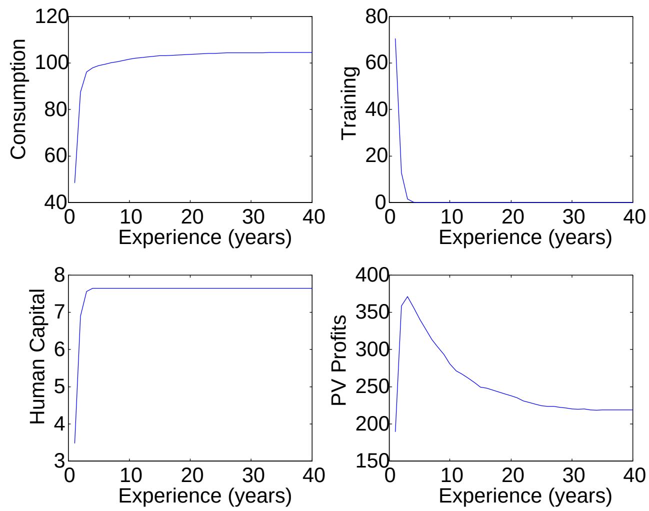
ployed. Both are higher with perfect commitment, reflecting the higher amount of resources devoted to training. Training is higher under perfect commitment because its return is higher, due to a lower probability of separation and an undistorted wage structure. As the last line in Table 3 shows, the range of the wage distribution is much larger with limited commitment, reflecting the need of firms to offer steep earnings profiles to satisfy worker´s participation constraints.

4.1.3 Simulating Time Profiles

Using the decision rules and distributions for the benchmark economy, we simulate work lives for a large number (1,000) of individuals as follows. We initialize each individual as an unemployed worker with the lowest level of human capital h_1 and let the economy run for a large number (1,000) of periods, using the decision rules and stochastic processes described above. Eventually, individuals find a job, receive training, are fired, and die, in which case they are replaced by newborn unemployed workers with the lowest human capital. We drop the first 100 periods to approximate the invariant distribution and end up with a panel of 1,000 workers and 900 periods (225 years).

Figure 5 presents the resulting tenure profiles for consumption (or wages), training, human capital and profits. In each case, we compute the average value of the variable across employed workers in our simulated panel with the same number of periods in the current match and same initial level of human capital when hired. As expected, for low skilled

Figure 6: Simulated Experience Profiles for the Average Worker



workers human capital and wages grow with tenure, while resources devoted to training fall over time (i.e., most of the training is done when the worker just joined the firm). The value of the match (present value of profits) also increases with tenure for these workers, since most of the cost of training is payed by the firm at the beginning. Workers hired as high skilled have flat tenure-consumption profiles. The value of the match for these workers is also constant, and lower on average than for low skilled workers, due to the higher value of new high skilled workers.

Figure 6 presents experience profiles for the same variables. In each case, we compute the average value of the variable across employed workers in our simulated panel with the same number of periods since born. Again, human capital and wages grow with experience. Interestingly, profits have a hump-shaped profile with respect to the experience of the worker. As shown in Figure 3, low skilled workers who increase their human capital inside the firm as a result of training are the ones who generate higher profits for such firm. On average, those workers are concentrated at mid-low levels of experience.

Using the results of the simulation, we compute the average tenure (or duration of employment spell) for workers of same level of initial human capital. The results are presented in Table 4. The average tenure is about 6 years in the model, while in the data for the U.S. it is slightly larger, about 7.4 years according to the Employment Outlook (2003). This reflects that the calibrated model generates too much job destruction, as discussed before. Also consistent with the evidence, high skilled workers have higher employment durations,

Table 4: Simulated Statistics for the Benchmark Economy

	Simulated Value
Average employment duration	
- low human capital	5.83
- high human capital	6.17
Mincerian returns to	
- experience	0.032
- tenure	0.021

which gives an additional incentive for training.

Finally, for our panel of workers we run two Mincerian earnings regressions:

$$\log(w_{it}) = a_0 + a_1 \times \exp + a_2 \times \exp^2 + \epsilon$$

and

$$\log(w_{it}) = b_0 + b_1 \times ten + b_2 \times ten^2 + \epsilon$$

We interpret the resulting coefficients a_1 and b_1 as the Mincerian returns to experience and tenure, respectively, and report them in Table 4. As in the empirical literature, returns to experience are larger than returns to tenure. Moreover, the value obtained for the return to experience (3.2%) is very much in line to the estimates of Lorenz and Wagner (1992) of the returns to experience for the U.S., controlling for differences in education.

4.2 Increasing the Firing Cost

The first policy experiment that we perform is to increase the firing cost, from zero in the benchmark economy to $\overline{f} = 200$. This value corresponds in equilibrium to about one year of the average worker's wage, and seems a reasonable number by European standards. The results of the experiment are presented in the second column of Table 5, which has to be compared to the first one (benchmark economy).

Firing costs have two important effects on the equilibrium: First, they reduce average profits, and hence the outside value of both types of workers. Second, they decrease the cut-off productivity, and hence increases the average length of tenure for both workers. The first effect directly tackles the commitment problem, reducing the incidence of participation constraints. The second effect increases the returns to training. Both effects move the economy in the direction of perfect commitment and, as a result, produce more training and

Table 5: Policy Experiments

	Benchmark Economy	Increase Firing Cost ($\overline{f} = 200$)	Increase Unem. Benef. ($\overline{b} = 50$)
Value unemployed worker			
- low human capital	279.3	278.2	280.6
- high human capital	284.7	283.8	287.8
Value new worker			
- low human capital	281.1	280.0	281.8
- high human capital	287.1	286.2	289.6
Threshold productivity			
- low human capital	13.93	12.86	15.00
- high human capital	12.23	11.03	12.84
Unemployment rate (percent)	5.48	5.39	9.02
Aggregate human capital			
- employed	6.35	6.44	6.02
- unemployed	0.31	0.31	0.48
Range of wage distribution			
- D9/D1 ratio	4.96	7.32	10.23
Avg. employment duration			
- low human capital	5.83	5.87	5.49
- high human capital	6.17	6.71	6.05
Mincerian returns to			
- experience	0.032	0.040	0.047
- tenure	0.021	0.027	0.031

higher human capital than in the benchmark economy.

The effect of firing costs on the unemployment rate is theoretically ambiguous. On one hand, firing costs reduce job creation by reducing the value of a vacancy. But on the other hand, firing costs reduce job destruction by reducing endogenous firing. The net effect in our numerical experiment is to decrease the unemployment rate, but by a very small amount.

Finally, firing costs have also two opposite effects on the steepness of the earnings profile, and then on the distribution of wages. First, as discussed before, firing costs mitigate the impact of participation constraints, allowing firms to provide for more insurance through a flatter wage profile. However, firing costs also reduce the average productivity of firms starting with low skilled workers, due to the usual selection effect: given a distribution of shocks, the cut-off productivity shifts to the left. Firms starting with high skilled workers are not subject in our model to productivity shocks, nor to the selection effect. This particular assumption of the model generates more wage inequality, and in our numerical experiment dominates the first effect.

4.3 Increasing Unemployment Benefits

We also perform the experiment of increasing unemployment benefits, from $\bar{b} = 22.8$ in the benchmark economy to $\bar{b} = 50$. The latter corresponds to an average unemployment benefit replacement rate of 54%, again a reasonable number for the most distorted European countries. The results are presented in the third column of Table 5, which again should be compared to the benchmark economy.

Unemployment benefits have exactly the opposite effect than firing cost on the incentives to train. First, they directly increase the outside value of both types of workers, by increasing the value of being unemployed. Second, they increase the cut-off productivity by shifting down the profit function, and therefore reduce the average tenure for both workers. By the opposite mechanism than the one described for firing costs, unemployment benefits decrease training and the average human capital of the economy.

Moreover, unemployment benefits unambiguously increase the unemployment rate, reducing job creation and, at the same time, increasing job destruction.

4.4 Sensitivity Analysis

(to be completed)

5 Conclusions

Training is a prevalent form of investment in human capital of the worker in all developed countries. The evidence also suggests that most forms of training are in general skills of the worker and are firm provided. However, while training is a prevalent form of investment in human capital, the intensity, structure, incidence, and effectiveness of training varies widely across countries.

We provide a unified framework to understand differences in training, unemployment, and wage profiles across countries. We focus on differences in labor market policies, but the model is also able to generates differences in training across countries form other institutional and technological factors. This is a first step for a serious quantitative analysis.

Our main result is that employment protection (or firing costs) might be a useful policy instrument to foster on-the-job human capital accumulation. However, this comes with a cost in terms of the average quality of the matches and wage inequality. We do not provide any welfare analysis, since we focus only on stationary equilibria, but we anticipate also welfare losses from such labor market interventions.

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Technical Appendix

A Characterizing the Optimal Contract

We analyze the optimal recursive contract given the probability p of finding a job, using equilibrium conditions 1, 2 and 3. First, we characterize the exit rule for all matches, and show that it satisfies a simple threshold productivity rule. Then, we start characterizing the contract between a firm and a worker with the highest level of human capital h_n, and find the corresponding outside value v_n^{un}. This turns out to be a simpler problem since the firm does not invest in training. Then we go backwards and characterize the contract and outside value of workers with human capital level j for $j \in \{n-1, n-2, ..., 1, 0\}$.

A.1 Exit Rule

As a first step, we characterize the exit rule for a match with state variables h_j, A, and v. The match is destroyed ($e = 1$) as long as

$$\Pi_j^{e=0}\left(A, v_j^{un}\right) < -\overline{f} \tag{2}$$

where $\Pi_j^{e=0}(A, v)$ solves the Bellman equation (1).

Lemma 1: *The value function* $\Pi_j^{e=0}(A, v)$ *is strictly increasing in* A and j*.* ***(to be proved)***

Lemma 1 implies that for each level of human capital h_j the exit rule (2) is characterized by a threshold productivity level $A_j^* \in \left[0, \overline{A}\right]$, satisfying

$$\Pi_j^{e=0}\left(A_j^*, v_j^{un}\right) = -\overline{f}$$

and such that the firm will operate if and only if $A \geq A_j^*$. Of course, we should impose

$$A_j^* = \begin{cases} 0, & \text{if } \Pi_j^{e=0}\left(0, v_j^{un}\right) > -\overline{f} \\ \overline{A}, & \text{if } \Pi_j^{e=0}\left(\overline{A}, v_j^{un}\right) < -\overline{f} \end{cases}$$

Moreover, the cutoff level A_j^* is non-increasing in j.

A.2 Workers with h_n

Consider a standing match with state variables h_n, A, and v. It is clear that $v \geq v_n^{un}$, otherwise the worker would have quitted in the previous period, and $A \geq A_n^*$. Using the definition of the continuation values for the firm and the worker, we can write the Bellman equation (1) for $j = n$ as:

$$\Pi_n^{e=0}(A, v) = \max_{c, v_n'} \left\{ AF(h_n) - c + \beta(1-\eta)(1-\gamma)\Pi_n^{e=0}(A, v_n') \right\}$$

$$s.t \qquad u(c) + \beta(1-\eta)\left[(1-\gamma)v_n' + \gamma v_n^{un}\right] = v,$$

$$v_n' \geq v_n^{un}.$$

We omit the non-negativity constraint for consumption, which will never bind in equilibrium.

Let's denote by η_1 and η_2 the Lagrange multipliers of the promise keeping constraint and the participation constraint, respectively. The first order conditions for this problem include:

$$\frac{\partial}{\partial c}: \qquad \eta_1 = \frac{1}{u'(c)},$$

$$\frac{\partial}{\partial v_n'}: \qquad \eta_2 = \beta(1-\eta)(1-\gamma)\left[\frac{1}{u'\left[c'(A, v_n')\right]} - \frac{1}{u'(c)}\right],$$

and the Kuhn-Tucker condition:

$$\eta_2 (v_n' - v_n^{un}) = 0.$$

The solution to the system implies a constant consumption profile for the worker ($c' = c$) and hence a constant promised value:

$$g_n^{v_n'}(A, v) = v.$$

With worker's risk-aversion, the least costly way of delivering an initial value v to the worker is through a smooth consumption profile. Note that, since $v \geq v_n^{un}$, the participation constraint for the worker is satisfied at each period.

Using the promise keeping constraint, we obtain:

$$g_n^c(A, v) = u^{-1}\left[\left(1 - \hat{\beta}\right)v - \tilde{\beta}v_n^{un}\right], \tag{3}$$

with $\hat{\beta} \equiv \beta(1-\eta)(1-\gamma)$ and $\tilde{\beta} \equiv \beta(1-\eta)\gamma$.

Finally, replacing in the value function for the firm:

$$\Pi_n^{e=0}(A,v) = AF(h_n) - u^{-1}\left[\left(1-\hat{\beta}\right)v - \tilde{\beta}v_n^{un}\right] + \hat{\beta}\Pi_n^{e=0}(A,v),$$

we obtain

$$\Pi_n^{e=0}(A,v) = \frac{AF(h_n) - u^{-1}\left[\left(1-\hat{\beta}\right)v - \tilde{\beta}v_n^{un}\right]}{1-\hat{\beta}}. \tag{4}$$

Now, we move to the Nash bargaining problem that determines the value of a new worker with human capital h_n. This value solves:

$$\max_v \left\{\Pi_n\left(\overline{A},v\right)^{\theta}\left(v - v_n^{un}\right)^{1-\theta}\right\},$$

with first order conditions:

$$\theta\frac{\partial\Pi_n}{\partial v}\left(\overline{A},v\right)\left(v - v_n^{un}\right) + (1-\theta)\,\Pi_n\left(\overline{A},v\right) = 0,$$

from which we obtain the sharing rule:

$$v_n^{new} = v_n^{un} + \left(\frac{1-\theta}{\theta}\right)\frac{\Pi_n\left(\overline{A},v_n^{new}\right)}{-\frac{\partial\Pi_n}{\partial v}\left(\overline{A},v_n^{new}\right)},$$

or, replacing (4) and its first derivative

$$v_n^{new} = v_n^{un} + \left(\frac{1-\theta}{\theta}\right)\frac{\overline{A}F(h_n) - u^{-1}\left[\left(1-\hat{\beta}\right)v - \tilde{\beta}v_n^{un}\right]}{\left(1-\hat{\beta}\right)\left(u^{-1}\right)'\left[\left(1-\hat{\beta}\right)v - \tilde{\beta}v_n^{un}\right]}. \tag{5}$$

Finally we determine the value of an unemployed worker with human capital h_n using:

$$v_n^{un} = u\left(\overline{b}\right) + \beta(1-\eta)\left[pv_n^{new} + (1-p)v_n^{un}\right],$$

from which:

$$v_n^{un} = \frac{u\left(\overline{b}\right) + \beta(1-\eta)\,pv_n^{new}}{1-\beta(1-\eta)(1-p)}. \tag{6}$$

The solution of the system of two equations (5) and (6) implicitly defines the values of an unemployed worker v_n^{un} and a new worker v_n^{new} as functions of p, $\overline{b}$, $\overline{f}$, and the other parameters of the model.

Finally, we come back to the exit rule, satisfying $\Pi_n^{e=0}(A, v_n^{un}) = -\overline{f}$. Given v_n^{un}, the threshold productivity level A_n^* solves

$$\frac{A_n^* F(h_n) - u^{-1}\left[(1-\beta(1-\eta))\, v_n^{un}\right]}{1-\hat{\beta}} = -\overline{f}$$

so that

$$A_n^* = \frac{u^{-1}\left[(1-\beta(1-\eta))\, v_n^{un}\right] - \left(1-\hat{\beta}\right)\overline{f}}{F(h_n)} \tag{7}$$

We can see as expected that an increase in the firing cost $\overline{f}$ decreases the threshold productivity level. We denote the policy rule for the exit decision:

$$g_n^e(A, v) = \begin{cases} 0, & \text{if } A \geq A_n^* \\ 1, & \text{if } A < A_n^* \end{cases} \tag{8}$$

A.3 Workers with $h_j < h_n$

Consider next a match with state variables $A \geq A_j^*$, h_j and $v \geq v_j^{un}$, for $j = 1, .., n-1$. Going backwards recursively from h_n, we already know the value for an unemployed worker with one additional unit of human capital, $\bar{v}_{j+1}^{un}$, the value function for the firm $\bar{\Pi}_{j+1}^{e=0}(A, v)$, the threshold productivity function $\overline{A}_{j+1}^*$ and the cumulative distribution function $G\left(\overline{A}_{j+1}^*\right)$. The optimal contract from that period onwards solves:

$$\Pi_j^{e=0}(A, v) = \max_{c,x,v_j',v_{j+1}'(A')} \Big\{ AF(h_j) - \psi x - c + (1-\rho(x))\,\hat{\beta}\,\Pi_j^{e=0}\left(A, v_j'\right)$$

$$+\rho(x)\,\hat{\beta}\left[\lambda \bar{\Pi}_{j+1}^{e=0}(A, v_{j+1}'(A)) + (1-\lambda)\left(\int_{\overline{A}_{j+1}^*}^{\overline{A}} \bar{\Pi}_{j+1}^{e=0}\left(A', v_{j+1}'(A')\right) dG(A') - G\left(\overline{A}_{j+1}^*\right)\overline{f}\right)\right]\Big\}$$

$$s.t \qquad u(c) + (1-\rho(x))\,\hat{\beta} v_j' + \widetilde{\beta}\left[(1-\rho(x))\, v_j^{un} + \rho(x)\,\bar{v}_{j+1}^{un}\right]$$

$$+\rho(x)\,\hat{\beta}\left[\lambda v_{j+1}'(A) + (1-\lambda)\left(\int_{\overline{A}_{j+1}^*}^{\overline{A}} v_{j+1}'(A')\, dG(A') + G\left(\overline{A}_{j+1}^*\right)\bar{v}_{j+1}^{un}\right)\right] = v,$$

$$v_j' \geq v_j^{un},$$

$$v_{j+1}'(A') \geq \bar{v}_{j+1}^{un}, \qquad \forall A' \in \left[0, \overline{A}\right].$$

with $\hat{\beta} \equiv \beta(1-\eta)(1-\gamma)$ and $\tilde{\beta} \equiv \beta(1-\eta)\gamma$. As before, we omit the non-negativity constraints on consumption and investment in training.

Denoting $\eta_1, \eta_{2,j}, \eta_{2,j+1}(A') \geq 0$ the Lagrange multipliers associated to each constraint, the first order conditions for this problem include:

$$\frac{\partial}{\partial c}: \qquad \eta_1 = \frac{1}{u'(c)},$$

$$\frac{\partial}{\partial v'_j}: \qquad \eta_{2,j} = \hat{\beta}(1-\rho(x))\left[\frac{1}{u'\left[c'\left(A, v'_j\right)\right]} - \frac{1}{u'(c)}\right],$$

$$\frac{\partial}{\partial v'_{j+1}(A')}: \qquad \eta_{2,j+1}(A') = \hat{\beta}(1-\lambda)\rho(x)\left[\frac{1}{u'\left[c'\left(A', v'_{j+1}(A')\right)\right]} - \frac{1}{u'(c)}\right],$$

if $A' \geq \overline{A}^*_{j+1}$, and

$$\frac{\partial}{\partial x}: \qquad \lambda\bar{\Pi}^{e=0}_{j+1}\left(A, v'_{j+1}(A)\right) + (1-\lambda)\left(\int_{\overline{A}^*_{j+1}}^{\overline{A}} \bar{\Pi}^{e=0}_{j+1}\left(A', v'_{j+1}(A')\right) dG(A') - G\left(\overline{A}^*_{j+1}\right)\overline{f}\right)$$

$$-\Pi^{e=0}_j\left(A, v'_j\right) + \frac{1}{u'(c)}\left[\lambda v'_{j+1}(A) + (1-\lambda)\left(\int_{\overline{A}^*_{j+1}}^{\overline{A}} v'_{j+1}(A')\, dG(A') + G\left(\overline{A}^*_{j+1}\right)\bar{v}^{un}_{j+1}\right)\right.$$

$$\left. -v'_j + \frac{\gamma}{1-\gamma}\left(\bar{v}^{un}_{j+1} - v^{un}_j\right)\right] = \frac{\psi}{\hat{\beta}\rho'(x)},$$

plus the complementary slackness conditions:

$$\eta_{2,j}\left(v'_j - v^{un}_j\right) = 0,$$

$$\eta_{2,j+1}(A')\left(v'_{j+1}(A') - \bar{v}^{un}_{j+1}\right) = 0, \qquad \forall A' \in \left[0, \overline{A}\right].$$

Note that to obtain these first order conditions we use the envelope condition:

$$\frac{\partial \Pi^{e=0}_j(A, v)}{\partial v} = -\eta_1 = -\frac{1}{u'(c)},$$

holding for all j.

As in the previous case, the solution to the system implies a constant consumption profile for the worker ($c' = c$) if training is not effective, hence a constant promised value:

$$g^{v'_j}_j(A, v) = v,$$

Also, for states in which training is successful and separation occurs, the participation constraint binds and

$$g_j^{v'_{j+1}(A')}(A,v) = \bar{v}_{j+1}^{un}, \quad \forall A' \in \left[0, \overline{A}_{j+1}^{*}\right]$$

We need to find now the promised value if training is effective and the match continues. For this, we guess and verify later than $g_j^{v'_{j+1}(A')}(A,v)$ is non-decreasing in A and v Then the participation constraint binds for small values of A' and does not bind for large values. This is:

$$\eta_{2,j+1}(A') > 0 \Rightarrow g_j^{v'_{j+1}(A')}(A,v) = \bar{v}_{j+1}^{un}, \qquad \forall A' \in [0, B_{j+1}(A,v)]$$

and

$$\eta_{2,j+1}(A') = 0 \Rightarrow g_j^{v'_{j+1}(A')}(A,v) \geq \bar{v}_{j+1}^{un}, \qquad \forall A' \in \left[B_{j+1}(A,v), \overline{A}\right]$$

where $B_{j+1}(A,v)$ is the maximum $B \in \left[0, \overline{A}\right]$ satisfying $g_j^{v'_{j+1}(B)}(A,v) = \bar{v}_{j+1}^{un}$. Of course, we should impose

$$B_{j+1}(A,v) = \begin{cases} 0, \text{ if } g_j^{v'_{j+1}(B)}(0,v) > \bar{v}_{j+1}^{un} \\ \overline{A}, \text{ if } g_j^{v'_{j+1}(B)}\left(\overline{A},v\right) = \bar{v}_{j+1}^{un} \end{cases}$$

If $\eta_{2,j+1}(A') = 0$ the firm offers a smooth consumption profile over states and over time:

$$g_j^c(A,v) = g_{j+1}^c\left(A', v'_{j+1}(A')\right)$$

Using the envelope condition and given a known function $\bar{\Pi}_{j+1}$, this implies

$$\frac{\partial \bar{\Pi}_{j+1}^{e=0}\left(A', v'_{j+1}(A')\right)}{\partial v} = -\frac{1}{u'\left(g_j^c(A,v)\right)}, \tag{9}$$

for all $A' \geq B_{j+1}(A,v)$.

Now, using $v'_j(A') = v$ and $v'_{j+1}(A') = \bar{v}_{j+1}^{un}$, $\forall A' \in [0, B_{j+1}(A,v)]$ we obtain the promise keeping constraint

$$\left[1 - (1-\rho(x))\hat{\beta}\right] v = u(c) + \widetilde{\beta}\left[(1-\rho(x)) v_j^{un} + \rho(x)\bar{v}_{j+1}^{un}\right]$$

$$+\rho(x)\hat{\beta}\left[\lambda v'_{j+1}(A) + (1-\lambda)\left(\int_{B_{j+1}(A,v)}^{\overline{A}} v'_{j+1}(A')\,dG(A') + G(B_{j+1}(A,v))\,\bar{v}_{j+1}^{un}\right)\right]. \tag{10}$$

Finally, we use the first order condition for training

$$\lambda\bar{\Pi}_{j+1}^{e=0}\left(A, v_{j+1}'(A)\right) + (1-\lambda)\left(\int_{\overline{A}_{j+1}^{*}}^{\overline{A}} \bar{\Pi}_{j+1}^{e=0}\left(A', v_{j+1}'(A')\right) dG(A') - G\left(\overline{A}_{j+1}^{*}\right)\overline{f}\right)$$

$$-\Pi_{j}^{e=0}(A,v)+\frac{1}{u'(c)}\left[\lambda v_{j+1}'(A) + (1-\lambda)\left(\int_{B_{j+1}(A,v)}^{\overline{A}} v_{j+1}'(A')\, dG(A') + G\left(B_{j+1}(A,v)\right)\bar{v}_{j+1}^{un}\right)\right.$$

$$\left. -v + \frac{\gamma}{1-\gamma}\left(\bar{v}_{j+1}^{un} - v_{j}^{un}\right)\right] = \frac{\psi}{\hat{\beta}\rho'(x)},$$

which, replacing the value function,

$$\Pi_{j}^{e=0}(A,v) = \frac{1}{1-\hat{\beta}\left(1-\rho(x)\right)}\left\{AF(h_j) - \psi x - c + \rho(x)\hat{\beta}\left[\lambda\bar{\Pi}_{j+1}^{e=0}(A, v_{j+1}'(A))\right.\right.$$

$$\left.\left. + (1-\lambda)\left(\int_{\overline{A}_{j+1}^{*}}^{\overline{A}} \bar{\Pi}_{j+1}^{e=0}\left(A', v_{j+1}'(A')\right) dG(A') - G\left(\overline{A}_{j+1}^{*}\right)\overline{f}\right)\right]\right\}, \tag{11}$$

becomes:

$$\frac{1}{1-\hat{\beta}\left(1-\rho(x)\right)}\left\{-AF(h_j) + \psi x + c + \left(1-\hat{\beta}\right)\left[\lambda\bar{\Pi}_{j+1}^{e=0}(A, v_{j+1}'(A))\right.\right.$$

$$\left.\left. + (1-\lambda)\left(\int_{\overline{A}_{j+1}^{*}}^{\overline{A}} \bar{\Pi}_{j+1}^{e=0}\left(A', v_{j+1}'(A')\right) dG(A') - G\left(\overline{A}_{j+1}^{*}\right)\overline{f}\right)\right]\right\}$$

$$+\frac{1}{u'(c)}\left[\lambda v_{j+1}'(A) + (1-\lambda)\left(\int_{B_{j+1}(A,v)}^{\overline{A}} v_{j+1}'(A')\, dG(A') + G\left(B_{j+1}(A,v)\right)\bar{v}_{j+1}^{un}\right) - v\right.$$

$$\left. + \frac{\gamma}{1-\gamma}\left(\bar{v}_{j+1}^{un} - v_{j}^{un}\right)\right] = \frac{\psi}{\hat{\beta}\rho'(x)}, \tag{12}$$

Then, equations (9), (10) and (12) define a system which implicitly determines $g_j^c(A,v)$, $g_j^x(A,v)$, and $g_j^{v_{j+1}'(\bullet)}(A,v)$ depending on $B(A,v)$. The complete fixed point problems involves using $v_{j+1}'(A,v)$ to determine $B(A,v)$.

Lemma 2: *The policy rule* $g_j^{v_{j+1}'(A')}(A,v)$ *is non-decreasing in* A *and* v. ***(to be proved)***

All these functions depend on v_j^{un} which is unknown. Moving to the Nash bargaining problem, we determine the value of a new worker with human capital h_j solving:

$$\max_{v}\left\{\Pi_j\left(\overline{A}, v\right)^{\theta}\left(v - v_j^{un}\right)^{1-\theta}\right\},$$

with first order conditions:

$$\theta\frac{\partial\Pi_j}{\partial v}\left(\overline{A},v\right)\left(v-v_j^{un}\right)+(1-\theta)\,\Pi_j\left(\overline{A},v\right)=0,$$

from which we obtain the sharing rule:

$$v_j^{new}=v_j^{un}+\left(\frac{1-\theta}{\theta}\right)\frac{\Pi_j\left(\overline{A},v_j^{new}\right)}{-\frac{\partial\Pi_j}{\partial v}\left(\overline{A},v_j^{new}\right)}. \tag{13}$$

and finally we determine the value of an unemployed worker with human capital h_n using:

$$v_j^{un}=u\left(\bar{b}\right)+\beta\left(1-\eta\right)\left[pv_j^{new}+(1-p)v_j^{un}\right],$$

from which:

$$v_j^{un}=\frac{u\left(\bar{b}\right)+\beta\left(1-\eta\right)pv_j^{new}}{1-\beta\left(1-\eta\right)\left(1-p\right)}. \tag{14}$$

The solution of the system of four equations (11), (13) and (14) implicitly defines the value $\Pi_j^{e=0}\left(\overline{A},v_j^{new}\right)$, the values of an unemployed worker v_n^{un} and a new worker v_j^{new} as functions of p, $\bar{b}$, $\bar{f}$, and the other parameters of the model.

Finally, given v_j^{un} we can obtain the cutoff level A_j^* and the exit rule $g_j^e\left(A,v\right)$ using equation (11) and the condition $\Pi_j^{e=0}\left(A,v_j^{un}\right)=-\overline{f}$.

B Computing a Stationary Equilibrium

We discretize the space for the state variables A, v constructing grids with n_A, n_v points respectively. Now, the equilibrium objects reduce to matrices of size $n_v\times n_A$ (Π_j, g_j^c, g_j^x, g_j^e $g_j^{v'_j}$), $n_v\times n_A\times n_A$ matrix$g_j^{v'_{j+1}(A')}$, $n\times n_v\times n_A$ matrix μ^{em} and vectors of size n (μ^{un}, v^{un} and v^{new}), in addition to the numbers u, s, p and q.

The algorithm to compute the equilibrium is based in a iteration procedure on the labor market tightness ratio u/s. This outer loop includes two inner loops: first, an iteration on the value of an unemployed worker v^{un}; second, an iteration on the invariant distributions μ^{un} and μ^{em}. The steps are as follows:

1. Guess an initial ratio u/s;

2. Given u/s, and using equilibrium condition 5, find probabilities p and q;

3. Given p, find values v_n^{new} and v_n^{un} solving the system (5) - (6);

4. Given v_n^{un}, find g_n^c from (3), Π_n using (4), A_n^* using (7), g_n^e from (8), and let $g_n^x \equiv 0$, $g_n^{v_j'} \equiv v$ and $g_n^{v_{j+1}'} \equiv 0$;

5. Given p, v_{j+1}^{un}, Π_{j+1}, and A_{j+1}^*, find Π_j, A_j^*, v_j^{un}, v_j^{new}, g_j^c, g_j^x, g_j^e, $g_j^{v_j'}$, and $g_j^{v_{j+1}'(\bullet)}$ for each $j \in \{1, ..., n-1\}$ by backward induction. At each human capital level h_j proceed as follows:

   a. Guess v_j^{un};

   b. Guess the policy rule g_j^c;

   c. Given $\bar{\Pi}_{j+1}$, $\bar{v}_{j+1}^{un}$, v_j^{un}, $g_j^{v_j'} = v$, and g_j^c, obtain in the following order:

      - the policy rule $g_j^{v_{j+1}'(A')}$ from envelope condition (9);
      - the cut-off $B(A, v)$ solving $g_j^{v_{j+1}'(B)} = \bar{v}_{j+1}^{un}$ and taking $B(A, v) = \max\left\{B, A_{j+1}^*\right\}$;
      - the policy rule g_j^x from (12);
      - the value function Π_j using (11);
      - the threshold value A_j^*, solving $\Pi_j(A, v_n^{un}) = -\overline{f}$;
      - the exit rule g_j^e using A_j^*.

   d. Check promise keeping constraint (10). If not satisfied, go back to step 5(b) and update the guess for g_j^c;

   e. Given v_j^{un} and Π_j, find v_j^{new} using (13);

   f. Check (14). If not satisfied, go back to step 5(a) and update the guess for v_j^{un}.

6. Given p, g_j^x, $g_j^{v_j'}$, and $g_j^{v_{j+1}'}$, find probability measures μ^{un} and μ^{em} as follows:

   a. Guess μ^{un} and μ^{em}

   b. Check equilibrium conditions 6 and 7. If not satisfied, go back to step 6(a) and update the guess for μ^{un} and μ^{em}

7. Given u/s, Π_j, v_j^{new}, and μ^{un}, check equilibrium condition 4. If not satisfied, go back to step 1 and update the guess for u/s.