ECO 426 (Market Design) - Lecture 2

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decentralized matching vs. NRMP

- In U.S., Canada and other countries, doctors work as "hospital residents" after completing medical school
- Before 1952 medical students found residencies through a decentralized process
- Main problem experienced was one of early contracting
  - hospital and medical students rushing to sign up residency contracts very early, for fear of not being able to form a match if waiting
  - costly in terms of match quality
- Other potential inefficiencies with decentralized matching
  - holding offers for long time and then rejecting after the market has cleared
  - exploding offers (i.e. offers with a very short deadline)

decentralized matching vs. NRMP

- Hospitals decided to change the system by adopting a central clearinghouse: National Residency Matching Program (NRMP)
  - medical students submit a list of preferences over hospitals
  - hospitals submit a list of preferences over students
  - the NRMP comes out with a matching on the basis of the inputed preferences
- It turns out that the NRMP uses the Gale-Shapley algorithm (since 1952)
- NRMP has been successful and persisted with few modifications, the stability property might help explain the program's success (Roth 1984)
  - The algorithm was changed in the late 90s from hospital proposing to students proposing

priority matching

- Alternative centralized matching mechanism: Priority Matching
  - agents submit preference lists
  - a pair of agents is given a "priority" depending on their mutual ranking
  - pairs with highes priority are formed first and agents taken out of the market
  - priorities are re-assigned after exit of agents and matching process continues...
- Example: priority equal to the product of mutual rankings (i.e. couples that rank each other first have highest priority, couples with mutual ranking 1-2 have second highest priority etc. etc.)

priority matching

- Priority Matching has been used in real life centralized matching
  - UK residency matching in Newcastle adopted in 1967
- Less successful than the NRMP
  - By the early 80s most participants in the Newcastle matching program were submitting just one choice (medical students and hospital were pre-contracting before formally participating in the matching program)
  - priority matching is no longer in use in Newcastle
- Is stability of the matching outcome important for the success of a centralized matching program?

TABLE I

STABLE AND UNSTABLE (CENTRALIZED) MECHANISMS

Market	Stable	Still in use (halted unraveling)
American medical markets		
NRMP	yes	yes (new design in '98)
Medical Specialties	yes	yes (about 30 markets)
British Regional Medical Markets		
Edinburgh ('69)	yes	yes
Cardiff	yes	yes
Birmingham	no	no
Edinburgh ('67)	no	no
Newcastle	no	no
Sheffield	no	no
Cambridge	no	yes
London Hospital	no	yes
Other healthcare markets		
Dental Residencies	yes	yes
Osteopaths (<'94)	no	no
Osteopaths (≥'94)	yes	yes
Pharmacists	yes	yes
Other markets and matching processes		
Canadian Lawyers	yes	yes (except in British Columbia since 1996)
Sororities	yes (at equilibrium)	yes

From: Roth, 2002, "The economist as engineer: game theory, experimentation, and computation as tools for economic design;" *Econometrica,* 1341--1378

Strategic incentives

- A centralized matching mechanism uses participants' preferences to determine a matching
- Preferences are subjective and private information to the participants
- Preference elicitation problem
  - Question: will the participants have the incentive to honestly reveal their preferences?
  - Answer: It depends on
    - the details of the centralized matching mechanism;
    - possibly, on the behavior of other participants.

Formal strategic model

- Formalize the preference elicitation problem as a strategic game with ordinal preferences
  - Players: the matching market participants - $M \cup W$
  - Actions: for each player the collection of all possible preference orderings (rank ordered lists (ROL)) over matches - Q^M and Q^W or Q
  - Outcomes: determined by the matching mechanism chosen, $\mathcal{H}$
    - A matching mechanism is a function that maps a profile of reported preferences q into a matching $\mathcal{H}(q)$
  - Preferences: for each player a "true" preference ordering over partners - P_m for a typical man m and P_w for a typical woman w, P for all players
- The triplet: i) $M \cup W$; ii) $\mathcal{H}$; and iii) P; define a strategic game with ordinal preferences.

Formal strategic model

- A strategy for a player m is a preference ordering over matches, $q_m \in Q^M$
- A strategy for a player w is a preference ordering over matches, $q_w \in Q^W$
- A strategy profile, q, is a collection of a strategy for each player
  - We use q_{-x} to denote a profile of strategies for all but player x
- Definition (Best response) A strategy q_x is a best response to a strategy profile profile q_{-x} for player x if the matching $\mathcal{H}(q_x, q_{-x})$ is (weakly) preferred by x to the matching $\mathcal{H}(q'_x, q_{-x})$ for any other strategy q'_x available to x.

Formal strategic model

- Definition (Dominant strategy) A strategy q_x is a (weakly) dominant strategy for player x if it is a best response to all possible strategy choices by the other players (i.e. it is a best response to all $q_{-x} \in Q_{-x}$.)
- Whether a player has a dominant strategy depends on the matching mechanism ($\mathcal{H}$), as well as on the agent's true preferences.
- If for a mechanism $\mathcal{H}$, P_x is a dominant strategy for player x, then player x has no reason (i.e. he/she never gains) to misreport her preferences within that mechanism.

desirable properties of a matching mechanism

- A matching mechanism is strategy proof if for every agent x, and for any profile of true preference, P_x is a dominant strategy.
  - In a strategy proof mechanism, preference elicitation is not a problem
  - Is there any strategy proof mechanism?
    - Yes. Example: random matching independent of reported preferences
    - But the outcome can be inefficient i.e. every agent could be made better off by some other matching

desirable properties of a matching mechanism

- A matching mechanism is Pareto efficient, if $\mathcal{H}(q)$ is a pareto efficient matching with respect to the preference profile q for any $q \in Q$.
  - If a matching mechanism is both strategy proof and pareto efficient, preference elicitation is not a problem and the outcome is pareto efficient (with respect to the true preferences)
  - Is there any strategy proof **and** pareto efficient mechanism?
    - Yes. Example: (random) serial dictatorship. Agents choose in a given order, each agent "picks" the favorite partner among those still available at moment of choosing.
    - Real life example: NFL draft
    - But the outcome can be **not stable**

desirable properties of a matching mechanism

- A matching mechanism is stable, if $\mathcal{H}(q)$ is a stable matching with respect to the preference profile q for any $q \in Q$.
  - If a matching mechanism is both strategy proof and stable, then preference elicitation is not a problem and the outcome is stable (and pareto efficient) with respect to the true preferences
  - Is there any strategy proof **and** stable mechanism? **No**

impossibility of strategy proof and stable mechanism

Example $M = \{m_1, m_2\}$ and $W = \{w_1, w_2\}$

m_1	w_1	w_2	w_1	m_2	m_1
m_2	w_2	w_1	w_2	m_1	m_2

- Two stable matching given the true preferences
  - $(m_1, w_1), (m_2, w_2)$ is favorite by men
  - $(m_1, w_2), (m_2, w_1)$ is favorite by women
- If m_1 claims that w_2 is an unacceptable partner (and everybody else reports honestly)

m_1	w_1		w_1	m_2	m_1
m_2	w_2	w_1	w_2	m_1	m_2

  the only stable matching is the one favorite by men $(m_1, w_1), (m_2, w_2)$
- If w_1 claims that m_1 is an unacceptable partner (and anybody else report honestly,) the only stable matching is the one favorite by women

impossibility of strategy proof and stable mechanism

Example $M = \{m_1, m_2\}$ and $W = \{w_1, w_2\}$

m_1	w_1	w_2
m_2	w_2	w_1

w_1	m_2	m_1
w_2	m_1	m_2

- Regardless of which stable matching $\mathcal{H}$ selects when everybody report their true preferences some agent has an incentive to deviate i.e. the mechanism cannot be strategy strategy proof if it is stable.
- Is the dominant strategy requirement too restrictive?
  - The example also shows that there is no Nash equilibrium where all players report their true preferences as long as the mechanism is stable
  - Relaxing the requirement that reporting the true preferences is a dominant strategy, and only requiring that everybody reporting honestly is a Nash equilibrium does not help