ECO426, Winter 2015
University of Toronto
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Assignment 1
DUE BEFORE 11:59PM ON WEDNESDAY FEBRUARY 11, 2015

Instructions: The assignment must be typed and submitted via email to ettore.damiano@utoronto.ca. You should present your arguments both clearly and concisely. I will use the following marking scheme: 30% of the marks are for clarity, 20% of the marks are for conciseness and 50% of the marks are for correctness. You can discuss the problems with other students in the class, however you **must** write your own solution. Also, if when solving a particular problem you received a significant amount of help from one of more student in the class, you should acknowledge it in an appropriately placed footnote.

Question 1
Provide an example of a marriage market with at least four (4) different stable matchings, and compare the preferences of each agent over the set of stable matchings.

Question 2
Consider a many-to-one matching model where firms have responsive preferences. Suppose the DA algorithm with firms proposing is used to match firms and workers (after treating each firm with multiple openings as multiple identical firms with one opening). Construct an example where a firm can gain by misreporting their preferences.

Question 3
Following up on our class discussion, in a house allocation with existing tenants model, consider the incentive of an agent to misreport his/her preferences. Can it ever be that misreporting the true preferences by some agent a induces an outcome such that : i) a does not change his/her assignment (i.e. a is assigned to the same house as when truthfully reporting); **and** ii) some other agent a' is assigned to a different house? If the answer is yes, provide an example, otherwise prove that it is not possible.

Question 4

Consider the following example of a marriage market with strict preferences. There are four men, $M = \{m_1, m_2, m_3, m_4\}$, and five women $W = \{w_1, w_2, w_3, w_4, w_5\}$. The preferences of each man and woman are described (in the form of ordered lists of acceptable mates) in the following two tables.

m_1	w_2	w_3	w_1	w_5	
m_2	w_3	w_2	w_5	w_1	w_4
m_3	w_1	w_2	w_3	w_4	w_5
m_4	w_5	w_3	w_2	w_1	w_4

w_1	m_2	m_1	m_3	m_4
w_2	m_4	m_3	m_2	m_1
w_3	m_3	m_4	m_1	
w_4	m_2	m_3	m_4	m_1
w_5	m_1	m_2	m_4	m_3

1) Using the Deferred Acceptance algorithm, find the men-optimal and the women -optimal stable matchings. (Describe each of the steps of the algorithm)

2) Find a stable matching different from those you found in 1). Compare the preferences of the agents across the three stable matchings you found.

3) The "Rural Hospital Theorem" states that the set of unmatched agents is the same across all stable matchings. Only using the fact that a men-optimal and a women-optimal stable matching exist, (i.e. among the stable matchings, there is one that is favored by all men, and one that is favored by all women,) demonstrate that the set of unmatched agents must be the same in the men- and women- optimal stable matchings.

Question 5

Consider a marriage market where matchings are formed using the DA women-proposing algorithm. However, there is an additional constraint that only matches among agents that have previously gone out on a date can be formed. You are asked to study the strategic form game where, before the DA algorithm is run, each man can ask exactly one woman on a date (no date request is refused).

1) Is the strategy profile where each man asks for a date with the woman he would be matched in the men-optimal stable matching a Nash equilibrium?

2) Is any other strategy profile a Nash equilibrium of the game? Provide an example, or prove that there is no Nash equilibrium candidate other than the strategy profile in part 1).